AG & LB Fysikum, Stockholms Universitet

Tentamen i Kosmologi och astropartikelfysik FK7007

9.00-14.00, 2012-05-30

Hjälpmedel: bifogad formelsamling, miniräknare och *Physics Handbook*.

1. a) Explain what is meant by the *critical energy density*, $\rho_c = \frac{3H^2}{8\pi G}$. What is its value today, i.e., ρ_c^0? (1p)
   b) Show that the Friedmann equation can be written as:

   $$H^2 = H_0^2 \left[\Omega_\gamma (1+z)^4 + \Omega_M (1+z)^3 + \Omega_K (1+z)^2 + \Omega_\Lambda\right],$$

   where $\Omega_M = \rho_M^0/\rho_c^0$,$\Omega_\gamma = \rho_\gamma^0/\rho_c^0$, $\Omega_K = -\frac{k}{a_0^2 H_0^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$
   All indices '0' indicate values for $z = 0$. (3p)

2. a) Show that an empty, flat universe with $\Lambda > 0$ expands as $a(t) \propto e^{Ht}$. (2p)

   b) Explain the relevance of this solution for the early universe and why an exponential expansion could solve some of the problems of the Big Bang model. (2p)

3. Neutrinos are kept in thermal equilibrium in the early universe through weak interactions, with typical cross-sections $\sigma = \frac{\alpha^2 T^2}{M_W^4}$, where α is the fine-structure constant ($\approx 1/137$) and $M_W \approx 80$ GeV/c^2 is the mass of the $W-$boson. The reaction rate is $\Gamma = n\sigma v$, where n and v are the neutrinos number density and average velocity.

   a) Explain why weak interactions are weaker than electromagnetic interactions and have short range. (1p)
   b) Explain the origin of a neutrino relic background. (1p)

   c) Show that neutrinos "decouple" at a temperature $T \approx \left(\frac{m_W^4}{\alpha^2 M_{Pl}}\right)^{\frac{1}{3}} \approx 4$ MeV. (Recall that $M_{Pl} = (\frac{\hbar c}{G})^{\frac{1}{2}}$). (2p)

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4. The number density of non-relativistic particles in thermal equilibrium is given by:

$$n_{NoRe} = g_i \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}}$$

   a) What are the interactions that can interchange neutrons and protons (i.e. start with one sort and end with the other)? (1p)
   b) Assuming a freeze out at T_f=0.8 MeV, estimate the relic abundance of ^{4}He (i.e., the baryon mass fraction in ^{4}He) assuming all neutrons are captured (2p)
   c) Explain the role of neutron life time and changes in T_f for the ^4He abundance. (1p)

5. Explain why the following measurements are sensitive to cosmological parameters.
   a) Dating of (old) stars and galaxies. (1p)
   b) Distance measurements to Type Ia supernovae. (1p)
   c) Strong gravitational lensing. (1p)
   d) Anisotropies in the Cosmic Microwave Background. (1p)

6. a) Assuming a constant equation of state parameter w and a single fluid in a flat universe and $a_0 = 1$, show that

$$a(t) = \left(\frac{3(1+w)}{2} H_0 t\right)^{\frac{2}{3(1+w)}}$$

   Hint: start from the fluid equation in the compilation of formulae (2p)
   b) Show that the deceleration parameter (q_0, also in the sheet) in such a universe becomes $q_0 = \frac{1+3w}{2}$. (2p)

Good luck!

Compilation of useful formulas

Equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}\ ; \qquad H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda}{3}; \qquad \dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

$$\frac{p}{c^2} = w \cdot \rho; \qquad q_0 = -\left(\frac{\ddot{a}(t_0)}{a(t_0)}\right)\frac{1}{H_0^2}$$

Constants

Newton's constant	G		6.672×10^{-11} m^3 kg^{-1} s^{-2}
Speed of light	c		2.998×10^{8} m s^{-1}
		or	3.076×10^{-7} Mpc year^{-1}
Planck's constant	$\hbar = h/2\pi$		1.055×10^{-34} m^2 kg s^{-1}
Boltzmann's constant	k_B		1.381×10^{-23} J K^{-1}
		or	8.619×10^{-5} eV K^{-1}
Radiation constant	$\alpha = \pi^2 k_B^4/15\hbar^3 c^3$		7.565×10^{-16} J m^{-3} K^{-4}
Electron mass	$m_e c^2$		0.511 MeV
Proton mass	$m_p c^2$		938.3 MeV
Neutron mass	$m_n c^2$		939.6 MeV
Planck mass	$M_{Pl} c^2$		$1.2 \cdot 10^{19}$ GeV
Thomson cross-section	σ_e		6.652×10^{-29} m^2
Neutron halftime (free neutron)	$t_{n,1/2}$		611 s
Hubble constant	H$_0$		100 h km s^{-1}Mpc^{-1}
		or	$\mathrm{H}_0^{-1} = 9.77\ \mathrm{h}^{-1} \times 10^9$ years
h			0.704 ± 0.025

Conversion factors

1 pc = 3.261 light-year = 3.086×10^{16} m

1 year = 3.156×10^{7} s

1 eV = 1.602×10^{-19} J

1 $M_\odot = 1.989 \times 10^{30}$ kg