RA, Fysikum, Stockholms universitet

Tentamen i Kosmologi och astropartikelfysik FK7007

9.00-14.00, 2013-06-05

Hjälpmedel: bifogad formelsamling, miniräknare och *Physics Handbook*.

Write clearly and motivate your answers!

1. a) Starting from the assumption that the overall size of the universe is given by the scale factor, $a(t)$, and that the position of a galaxy relative an observer is $r(t)$, derive the *Hubble law*, $v = H_0 r$. (1p)

   b) Show that the general expression of the physical distance,

   $$r = c \int_0^z \frac{dz'}{H(z')} \, ,$$

   to a galaxy at redshift, z can be written as the *Hubble law* in the low redshift limit. (2p)

2. a) A long, long time ago in a galaxy far, far away ($z = 1.702$), the Millenium Falcon is traveling away from the centre of the galaxy with the speed $0.759c$, attempting to escape an evil imperial battle cruiser. The Falcon emits an isotropic, ultra-luminous monochromatic distress signal at a wavelength of 1216 Å. At what wavelength will we observe the signal on Earth today? You may assume that the space ship is located between the core of its galaxy and us, and that both systems are at rest w.r.t. the CMB. (2p)

   b) The Milky Way is moving with a speed of $\sim 500\,\mathrm{km/s}$ with respect to the CMB. Assuming that this is typical for all galaxies, calculate the redshift for which peculiar velocities are $< 10\%$ of the measured redshift. This defines the so called *Hubble flow*. (1p)

   c) Why is $\rho_\gamma \propto (1+z)^4$ when $\rho_M \propto (1+z)^3$? What is the physical interpretation of this? (1p)

3. a) Cosmological parameters can be measured by studying how the positions of galaxies on the sky are correlated. A method often referred to as *Baryon Accoustic Oscillations*. Describe, in words, why this signal exists, and why it can be used to study cosmological parameters. (2p)

   b) Describe two ways the Hubble constant can be measured. (2p)

Turn page! →

4. The CMB temperature today is $T = 2.73\,\mathrm{K}$.

   a) Assume that the ionization energy of hydrogen was lower than 13.6 eV in the early universe. What implication would this have for the first peak of the CMB power spectrum? Motivate! (2p)

   b) At what redshift would the universe have become transparent to light if the baryon-to-photon ratio, $\eta_B = n_B/n_\gamma \sim 1$. The ionization energy for neutral hydrogen is 13.6 eV. (2p)

   c) What will the temperature of the CMB be in 10 Gyr. You may assume that the universe is Λ-dominated (2p)

5. Neutrinos are decoupled when the temperature in the early universe drops below $\sim 4\,\mathrm{MeV}$. Immediately after decoupling $T = T_\nu = T_\gamma$. The neutrinos are still relativistic and their temperature will continue to fall as the universe expands.

   At $T \geq 1\,\mathrm{MeV}$ the relativistic particles in thermal equilibrium are $e^\pm$, γ. At $T < 1\,\mathrm{MeV}$ only the photons are left. Show that the relation between T_ν and T_γ today is

$$T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma\,.$$

   (3p)

6. Explain the following
   a) The Sachs-Wolfe effect (1p)
   b) The geodesic equation (1p)
   c) Critical density (1p)
   d) Einstein radius (1p)

Good luck!

Compilation of useful formulas

Equations

$$\frac{\lambda_0}{\lambda_e} = \gamma \left(1 - \frac{v}{c}\cos\theta\right)$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3} =$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$= H_0^2\left[\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda}{3};$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

$$\frac{p}{c^2} = w \cdot \rho;$$

$$q_0 = -\left(\frac{\ddot{a}(t_0)}{a(t_0)}\right)\frac{1}{H_0^2}$$

$$n_{\mathrm{NoRe}} = g_i\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}}$$

$$n_{\mathrm{Re}} = \begin{cases} \frac{\zeta(3)}{\pi^2} g_i T^3 & \text{Bose Einstein} \\ \frac{3}{4}\left(\frac{\zeta(3)}{\pi^2} g_i T^3\right) & \text{Fermi Dirac} \end{cases}$$

$$g_{\mathrm{eff}}^s = \sum\nolimits_{i=\mathrm{bosons}} g_i\left(\frac{T_i}{T}\right) + \frac{7}{8}\sum\nolimits_{j=\mathrm{fermions}} g_j\left(\frac{T_j}{T}\right)^3$$

Constants

Newton's constant	G		6.672×10^{-11} m^3 kg^{-1} s^{-2}
Speed of light	c		2.998×10^8 m s^{-1}
		or	3.076×10^{-7} Mpc year^{-1}
Planck's constant	$\hbar = h/2\pi$		1.055×10^{-34} m^2 kg s^{-1}
Boltzmann's constant	k_B		1.381×10^{-23} J K^{-1}
		or	8.619×10^{-5} eV K^{-1}
Radiation constant	$\alpha = \pi^2 k_B^4/15\hbar^3 c^3$		7.565×10^{-16} J m^{-3} K^{-4}
Electron mass	$m_e c^2$		0.511 MeV
Proton mass	$m_p c^2$		938.3 MeV
Neutron mass	$m_n c^2$		939.6 MeV
Planck mass	$M_{Pl} c^2$		$1.2 \cdot 10^{19}$ GeV
Thomson cross-section	σ_e		6.652×10^{-29} m^2
Neutron halftime (free neutron)	$t_{n,1/2}$		611 s
Hubble constant	H$_0$		100 h km s^{-1}Mpc^{-1}
		or	H$_0^{-1}$ = 9.77 h^{-1} $\times 10^9$ years
h			0.704 ± 0.025
Cosmological constant	Λ		10^{-35} s^{-2}, 10^{-47} GeV^4, 10^{-29} g/cm^3

Conversion factors

1 pc = 3.261 light-year = 3.086×10^{16} m
1 year = 3.156×10^7 s
1 eV = 1.602×10^{-19} J
1 K = 8.62×10^{-5} eV
1 $M_\odot$ = 1.989×10^{30} kg