RA, Fysikum, Stockholms universitet

Tentamen i Kosmologi och astropartikelfysik FK7007

9.00-14.00, 2013-06-04

Hjälpmedel: bifogad formelsamling, miniräknare och *Physics Handbook*.

Write clearly and motivate your answers!

1. List two observations that support the Big Bang model. (1p)

2. The rotation curves, $v(r)$, for spiral galaxies have been observed to be close to flat for large radii. Using a test particle m, show that this is unexpected for radii $r > r_{\rm vis}$, where $r_{\rm vis}$ contains most of the visible matter, $M_{\rm vis}$. (2p)

3. Show that $|\Omega_k| = |k/(aH)^2|$ will grow with time if $k \neq 0$ and the universe is either radiation or matter dominated, assuming that $a \propto t^{\frac{2}{3(1+w)}}$. (2p)

4. a) Explain the meaning of the Jeans length,

$$\lambda_J = \sqrt{\frac{\pi}{G\rho}} v_s$$

where, v_s, is the sound speed. (1p)

b) What happens with the Jeans length for baryons after baryon-photon decoupling? (1p)

c) If a significant fraction of the dark matter is hot (relativistic), e.g. in the form of neutrinos, how would the Jeans length be affected? What would the qualitative implication be on the matter power spectrum? (2p)

5. Show that the Hubble parameter, $H(z)$, can be written as

$$H(z) = H_0 \left[(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda\right]^{1/2} .$$

(1p)

6. Photons decouple at the temperature of $T \sim 0.25\,\text{eV}$. Carry out an order-of-magnitude calculation of the neutrino decoupling temperature, assuming $\sigma_{\rm weak} \sim \alpha^2 T^2/m_W^4$, with the fine-structure constant $\alpha = 1/137$. The expansion rate of the early universe is $H \sim T^2/m_{\rm Pl}$. (2p)

7. Calculate the Christoffel symbol ${\Gamma^0}_{11}$ in an FLRW universe. Recall that

$$ {\Gamma^\sigma}_{\mu\nu} = \frac{g^{\rho\sigma}}{2}\left[\frac{\partial g_{\nu\rho}}{\partial x^\mu} + \frac{\partial g_{\mu\rho}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho}\right] .$$

(2p)

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8. a) The luminosity distance, d_L, can be written as

$$d_L = a \cdot r \cdot (1+z) = (1+z) \int\limits_0^z \frac{dz'}{H(z')}$$

for a flat universe. Using the FLRW line-element, show that the angular distance, $D = d_A \cdot \delta\theta$, can be written as $d_A = d_L/(1+z)^2$, with D being the distance subtending the angle $\delta\theta$ on the sky. *Hint:* Remember that the sign of the line-element is arbitrary. (3p)

b) For a universe with $\Omega_M = 1$ and $\Omega_\Lambda = 0$, show that the ratio $d_A(z=1)/d_A(z=0.5) > 1$ while $d_A(z=3)/d_A(z=1) < 1$. (2p)

c) Explain, in words, what the result in b) predicts for galaxies with the same physical size observed at the three different redshifts. (1p)

9. Briefly describe four different methods to measure cosmological parameters (H_0, Ω_M, etc.). (4p)

Good luck!

Compilation of useful formulas

Equations

$$\frac{\lambda_0}{\lambda_e} = \gamma\left(1 - \frac{v}{c}\cos\theta\right)$$

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3} = \qquad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$= H_0^2\left[\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda\right]$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda}{3}; \qquad \dot\rho + 3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

$$\frac{p}{c^2} = w\cdot\rho; \qquad q_0 = -\left(\frac{\ddot a(t_0)}{a(t_0)}\right)\frac{1}{H_0^2}$$

$$n_{\mathrm{NoRe}} = g_i\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}} \qquad n_{\mathrm{Re}} = \begin{cases} \frac{\zeta(3)}{\pi^2} g_i T^3 & \text{Bose Einstein} \\ \frac{3}{4}\left(\frac{\zeta(3)}{\pi^2} g_i T^3\right) & \text{Fermi Dirac} \end{cases}$$

$$g^s_{\mathrm{eff}} = \sum\nolimits_{i=\mathrm{bosons}} g_i\left(\frac{T_i}{T}\right) + \frac{7}{8}\sum\nolimits_{j=\mathrm{fermions}} g_j\left(\frac{T_j}{T}\right)^3$$

Constants

Newton's constant	G		6.672×10^{-11} m^3 kg^{-1} s^{-2}
Speed of light	c		2.998×10^{8} m s^{-1}
		or	3.076×10^{-7} Mpc year^{-1}
Planck's constant	$\hbar = h/2\pi$		1.055×10^{-34} m^2 kg s^{-1}
Boltzmann's constant	k_B		1.381×10^{-23} J K^{-1}
		or	8.619×10^{-5} eV K^{-1}
Radiation constant	$\alpha = \pi^2 k_B^4/15\hbar^3c^3$		7.565×10^{-16} J m^{-3} K^{-4}
Electron mass	m_ec^2		0.511 MeV
Proton mass	m_pc^2		938.3 MeV
Neutron mass	m_nc^2		939.6 MeV
W boson mass	m_Wc^2		80 GeV
Z boson mass	m_Zc^2		91 GeV
Planck mass	$M_{Pl}c^2$		$1.2\cdot10^{19}$ GeV
Thomson cross-section	σ_e		6.652×10^{-29} m^2
Neutron halftime (free neutron)	$t_{n,1/2}$		611 s
Hubble constant	H_0		100 h km s^{-1}Mpc^{-1}
		or	$\mathrm{H}_0^{-1} = 9.77\ \mathrm{h}^{-1}\times10^9$ years
h			0.704 ± 0.025
Cosmological constant	Λ		$10^{-35}\,\mathrm{s}^{-2}$, $10^{-47}\,\mathrm{GeV}^4$, $10^{-29}\,\mathrm{g/cm}^3$

Conversion factors

1 pc = 3.261 light-year = 3.086×10^{16} m
1 year = 3.156×10^{7} s
1 eV = 1.602×10^{-19} J
1 K = 8.62×10^{-5} eV
1 $M_\odot$ = 1.989×10^{30} kg