RA, Fysikum, Stockholms universitet

Tentamen i Kosmologi och astropartikelfysik FK7007
9.00-14.00, 2015-06-03

Hjälpmedel: bifogad formelsamling, miniräknare och *Physics Handbook*.

Write clearly and motivate your answers!

1. A two-dimensional space, parametrized by (r, θ), where θ runs between 0 and 2π, has the following metric

$$ds^2 = dr^2 + a^2 \sin^2\left(\frac{r}{a}\right) d\theta^2$$

where a is a constant.

a) What space does this metric describe? Motivate your answer. (2p)

b) If a circle with radius R is drawn on this space, what is its circumference?(1p)

c) What is the area of a circle of radius R on this space? (1p)

2. a) Show that the parametrization

$$\begin{aligned} a(\theta) &= \alpha\,(1-\cos\theta)\ , \\ t(\theta) &= \frac{\alpha}{\sqrt{k}}\,(\theta-\sin\theta)\ , \end{aligned}$$

where the parameter θ runs from 0 to 2π, describes a matter-dominated universe with positive curvature. Determine the value of the constant α. (2p)

b) Describe, in words, the state of the universe at $\theta = \pi$ and 2π. (1p)

3. a) What is the origin of the Baryon Acoustic Oscillations (BAO)? (1p)

b) Assuming that the Universe was radiation-dominated until decoupling took place at 400 000 years ($z \sim 1000$) after the Big Bang, show that the scale of the first BAO peak today is ~ 100 Mpc. (2p)

4. a) What is the quark compositions of a proton and a neutron? (0.5p)

b) What does a free neutron decay into? Describe how this involves weak interactions? (0.5p)

c) The mass difference between neutrons and protons is $\Delta m = 1.29$ MeV. Give an approximate value for the neutron to proton ratio at the "freeze-out" of weak interactions, $T = 0.8$ MeV. (1p)

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d) Assume all available neutrons are captured into ^4He. What is the mass abundance of Helium in the universe? (1p)

5. a) It may be expected that the oldest relic radiation is made of gravitons, the carriers of gravitation. Their interaction with other particles should have cross-section related to the Planck mass, $\sigma \sim \frac{T^2}{M_{\rm Pl}{}^4}$. Estimate the decoupling temperature of gravitons, T_g, under the assumptions $H \sim T^2/M_{\rm Pl}$ and $n \sim T^3$ $(c = k_B = 1)$. (2p)

   b) Assume that the present temperature of the relic radiation is ~ 1 K. At what redshift did the graviton decoupling take place? (1p)

6. a) The luminosity distance, d_L, is given by

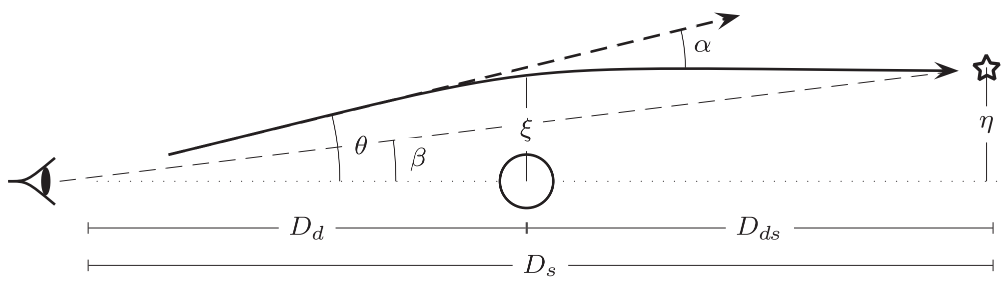
$$d_L = c\frac{1+z}{\sqrt{|\Omega_K|}}\mathcal{S}\left(\sqrt{|\Omega_K|}\int_0^z \frac{dz'}{H(z')}\right),$$

   where $\mathcal{S}(x)$ is defined as $\sin(x)$, x or $\sinh(x)$ depending on the geometry of the universe. Calculate the luminosity distances, in Mpc, to two galaxies at $z_d = 0.444$ and $z_s = 2.379$, respectively, in a flat universe with $\Omega_\Lambda = 0$ and $H_0 = 70$ km/s/Mpc. (2p)

   b) A gravitational lens system can be approximated by the *lens equation*

$$\theta = \beta + 2\frac{r_s}{\theta}\left(\frac{D_{ds}}{D_s \cdot D_d}\right) \quad \text{where} \quad r_s = \frac{2G_N M}{c^2}.$$

   The angles β and θ, and the distances D_d, D_s and D_{ds} are defined in the figure below. A distant galaxy ($z_s = 2.379$) is lensed by a foreground galaxy ($z_d = 0.444$) of mass M and forms an Einstein ring of $5''$ radius. Estimate the mass of the foreground galaxy using the assumptions in a) and that $D_{ds} \approx D_d$. (2p)



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7. Assume that the dark matter is made of elementary particles. a) Give two examples of observations that suggest the existence of dark matter. (1p)

   b) What is the difference between hot and cold dark matter? How do the two kinds affect structure formation? (1p)

   c) How does the relic density of dark matter particles depend on the annihilation cross-section? (1p)

   d) Give an example of a cold dark matter candidate that can be motivated by particle-physics. (1p)

Good luck!

Compilation of useful formulas

Equations

$\frac{\lambda_0}{\lambda_e} = \gamma\left(1 - \frac{v}{c}\cos\theta\right)$

$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3} =$

$= H_0^2\left[\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda\right]$

$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda}{3};$

$\frac{p}{c^2} = w \cdot \rho;$

$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$

$q_0 = -\left(\frac{\ddot{a}(t_0)}{a(t_0)}\right)\frac{1}{H_0^2}$

$n_{\rm NoRe} = g_i\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}}$

$$n_{\rm Re} = \begin{cases} \frac{\zeta(3)}{\pi^2} g_i T^3 & \text{Bose Einstein} \\ \frac{3}{4}\left(\frac{\zeta(3)}{\pi^2} g_i T^3\right) & \text{Fermi Dirac} \end{cases}$$

$g^s_{\rm eff} = \sum_{i=\rm bosons} g_i\left(\frac{T_i}{T}\right) + \frac{7}{8}\sum_{j=\rm fermions} g_j\left(\frac{T_j}{T}\right)^3$

Constants

Newton's constant	G_N		6.672×10^{-11} m^3 kg^{-1} s^{-2}
Speed of light	c		2.998×10^{8} m s^{-1}
		or	3.076×10^{-7} Mpc year^{-1}
Planck's constant	$\hbar = h/2\pi$		1.055×10^{-34} m^2 kg s^{-1}
Boltzmann's constant	k_B		1.381×10^{-23} J K^{-1}
		or	8.619×10^{-5} eV K^{-1}
Radiation constant	$\alpha = \pi^2 k_B^4/15\hbar^3 c^3$		7.565×10^{-16} J m^{-3} K^{-4}
Electron mass	$m_e c^2$		0.511 MeV
Proton mass	$m_p c^2$		938.3 MeV
Neutron mass	$m_n c^2$		939.6 MeV
W boson mass	$m_W c^2$		80 GeV
Z boson mass	$m_Z c^2$		91 GeV
Planck mass	$M_{Pl} c^2$		$1.2 \cdot 10^{19}$ GeV
Thomson cross-section	σ_e		6.652×10^{-29} m^2
Neutron halftime (free neutron)	$t_{n,1/2}$		611 s
Hubble constant	H_0		100 h km s^{-1}Mpc^{-1}
		or	$H_0^{-1} = 9.77$ h^{-1} $\times 10^9$ years
h			0.704 ± 0.025
Cosmological constant	Λ		10^{-35} s^{-2}, 10^{-47} GeV^4, 10^{-29} g/cm^3

Conversion factors

1 pc = 3.261 light-year = 3.086×10^{16} m
1 year = 3.156×10^{7} s
1 eV = 1.602×10^{-19} J
1 K = 8.62×10^{-5} eV
1 $M_\odot = 1.989 \times 10^{30}$ kg