INTRODUCTION TO MODERN COSMOLOGY

- (~ Dodelson Chapter 1)

Cosmology is really about answering the 3 oldest questions:

- Who are we? ⟶ What is the Universe made of / how did the elements form?
- Where do we come from? ⟶ How was the Universe born?
- Where are we going? ⟶ How did the Universe evolve, what is its fate?

Modern cosmology: quantitative, data-driven answers to these questions. Huge amount of data ($\sim 10^7$ galaxies, 10^8 pixels) to test against predictions. Data-driven science, no longer on the realm of "philosophy".

The expanding Universe

Universe is expanding ⟶ distances between galaxies increase

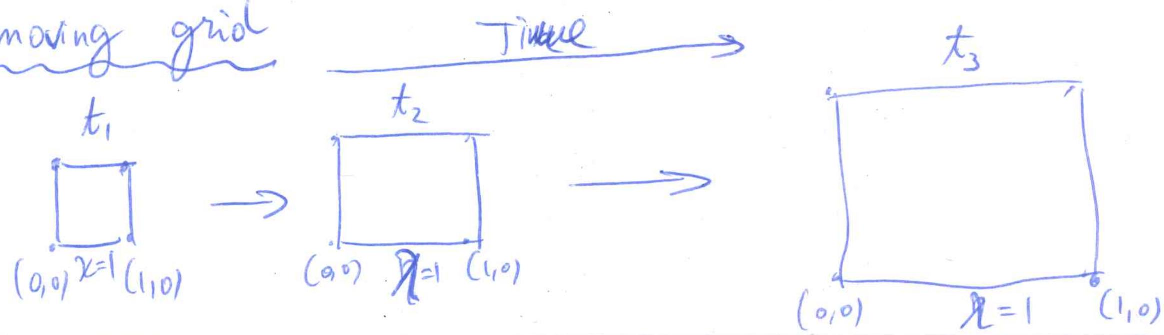
Scale factor a Today $a_0 = 1$, in the past $a < 1$

"0": today

Physical distances $\propto a$

Comoving distances: fixed

Comoving grid

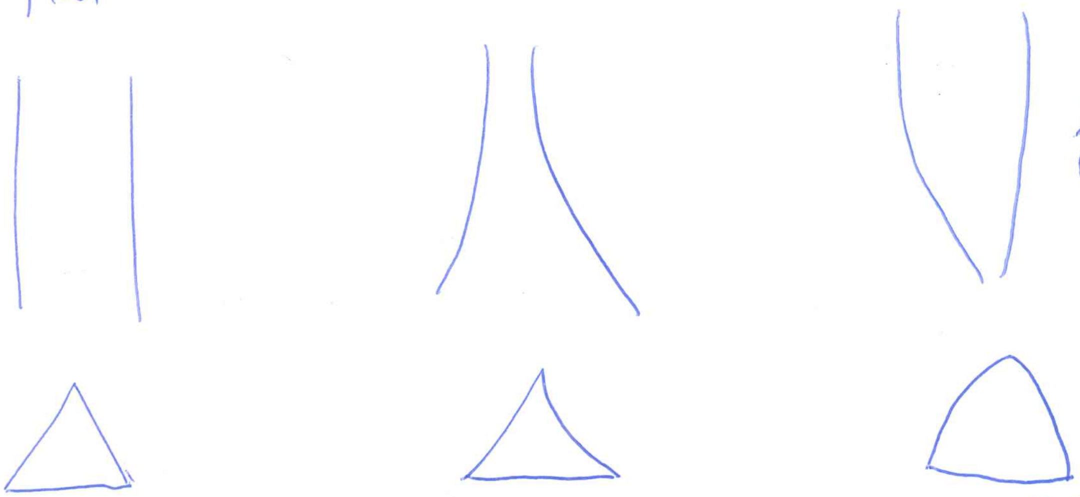


	t_1	t_2	t_3
Comoving distances:	$\chi = 1$	$\chi = 1$	$\chi = 1$
Physical distances	$a(t_1)\chi$	$a(t_2)\chi$	$a(t_3)\chi$

Smooth, expanding Universe characterized by ~~sa~~

<u>scale factor $a(t)$</u> <u>Geometry</u>

Flat Open Closed



Paths of free particles starting parallel to each other

180° $<180°$ $>180°$

no curvature negative curvature positive curvature

GR: Geometry $\equiv$ Energy

Flat Universe requires very special value for energy density today: $\rho_{crit} \sim 10^{-29} \frac{g}{cm^3}$

$\rho > \rho_{crit} \rightarrow$ closed

$\rho < \rho_{crit} \rightarrow$ open

GR tells us not only how energy and geometry of the Universe are related, but in turn how a evolves in time

$a(t)$ depends on the dominant component of energy

Radiation (γ, massless ν, etc. $P = \frac{\rho}{3}$) $\rightarrow a \propto t^{1/2}$

~~Matter~~
Non-relativistic matter ("dust", $P \sim 0$) $\rightarrow a \propto t^{2/3}$

Dark energy? ($P \sim -\rho$) $\rightarrow a \propto e^{t}$

$$\text{Geometry} \xleftarrow{GR} \text{Energy contents} \xrightarrow{GR} a(t)$$

Hubble rate (logarithmic derivative of scale factor)

$$\boxed{H(t) \equiv \frac{1}{a}\frac{da}{dt}} = \frac{\dot{a}}{a}$$

Radiation domination: $a \propto t^{1/2} \longrightarrow H = \frac{1}{2t}$

Matter domination : $a \propto t^{2/3} \longrightarrow H = \frac{2}{3t}$

Dark energy domination : $a \propto e^{t} \longrightarrow H = \text{const}$ $\quad (a \propto e^{Ht})$

$H_0 t_0$ powerful test of cosmology!
$\uparrow$ age of Universe today

First Friedmann equation gives evolution of scale factor
(you'll see it again later, no need to worry now)

$$\boxed{H^2(t) = \frac{1}{a^2}\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\left[\rho(t) + \frac{\rho_{cr} - \rho_0}{a^2(t)}\right]} \qquad \rho_{cr} \equiv \frac{3H_0^2}{8\pi G}$$

So the question of how $a(t)$ evolves boils down to figuring out / providing a prescription for $\rho(t)$

Non-relativistic matter ("dust"): "baryons", dark matter (DM)

$E \propto m \qquad \rho \propto \frac{E}{V} \propto mn \propto a^{-3}$ since $V \propto a^3$

$\rho_m \propto a^{-3}$

Radiation

$E \propto \frac{1}{\lambda} \propto \frac{1}{a} \longrightarrow T(t) = \frac{T_0}{a(t)}$

$E \to k_B T_0$, $\lambda \to \frac{hc}{k_B T_0}$

Temperature of a thermal bath of ~~ph~~ radiation (e.g. γ)

$\rho \propto \frac{E}{V} \propto \frac{1}{a}\frac{1}{a^3} \propto a^{-4}$ [$\rho \propto T^4$ Stefan-Boltzmann law]

$\rho_r \propto a^{-4}$

Dark energy

Cosmological constant Λ : $\rho_\Lambda \propto$ const even as space expands

Example of getting $a(t)$

Consider flat Universe

$H^2(t) = \frac{8\pi G}{3}\rho(t) \longrightarrow$ Radiation $\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 \propto a^{-4}$

$\rightarrow \frac{1}{a}\frac{da}{dt} \propto a^{-2} \longrightarrow \frac{da}{dt} \propto a^{-1} \longrightarrow \int da\, a \propto \int dt \longrightarrow t \propto a^2 \rightarrow a \propto t^{1/2}$

Matter $\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 \propto a^{-3} \rightarrow \frac{1}{a}\frac{da}{dt} \propto a^{-3/2} \rightarrow \frac{da}{dt} \propto a^{-1/2} \rightarrow \int da\, a^{1/2} \propto \int dt$

$\rightarrow t \propto a^{3/2} \rightarrow a \propto t^{2/3}$

Dark energy (cosmological constant)

$$\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 = \text{const} = H_0^2 \qquad \frac{1}{a}\frac{da}{dt} = H_0 \qquad \int\frac{da}{a} = H_0\int dt$$

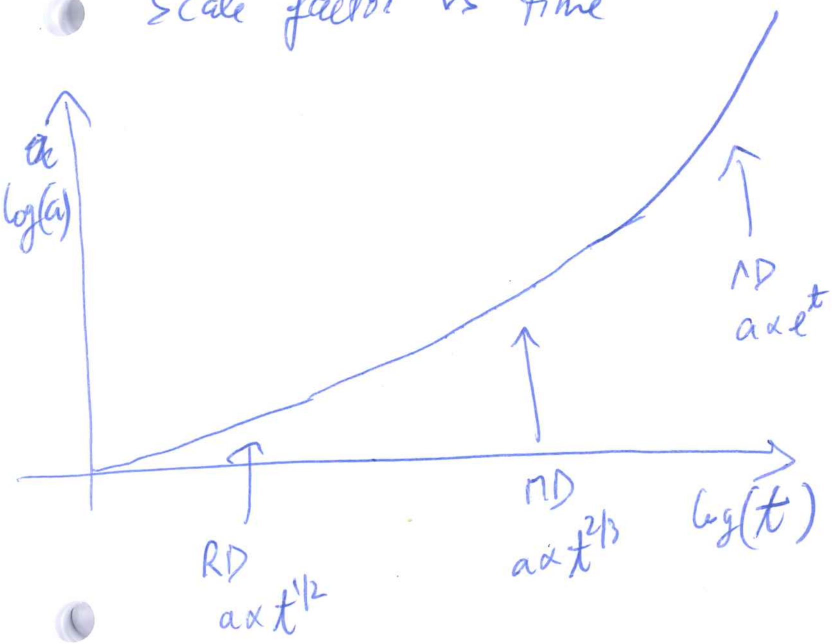
$$\ln a \propto H_0 t \rightarrow a \propto e^{H_0 t}$$

Recap

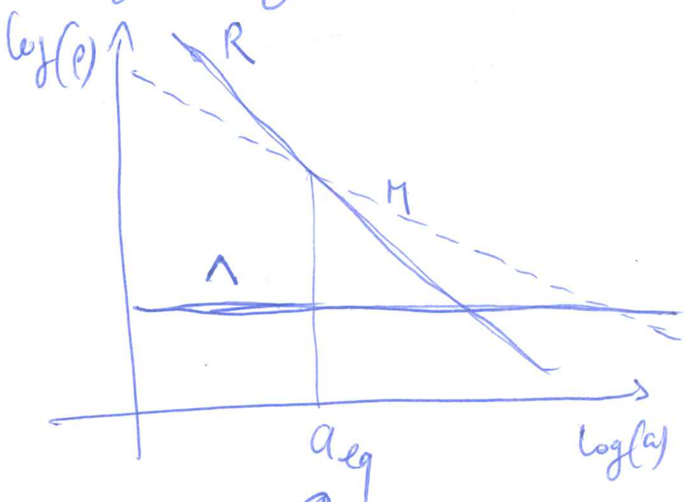
$$\rho_r \propto a^{-4} \rightarrow \rho_m \propto a^{-3} \longrightarrow \rho_\Lambda \propto \text{const}$$

[curvature effectively behaves as $\rho_k \propto a^{-2}$]

Scale factor vs time



Energy density vs scale factor



Epoch of matter-radiation equality

Some numbers

$$H_0 \sim 70 \frac{km}{s \cdot Mpc} \equiv 100\,h \frac{km}{s\cdot Mpc}$$

$Mpc = 10^6\,pc \sim 3.1 \times 10^{24}\,cm$

$h \sim 0.7$ $\left(H_0 \sim 70 \frac{km}{s \cdot Mpc}\right)$

$t_0 \sim 13$ Gyr

Easy to show $\underbrace{H_0 t_0}_{cH_0 t_0} > \frac{2}{3}$

Redshift and Hubble's law

Expanding Universe → galaxies receding → wavelength of emitted light stretched

Stretching factor : $1+z$ ← redshift

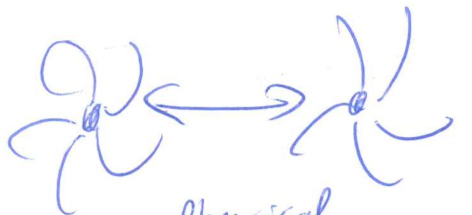
$$1+z \equiv \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{1}{a} > 1$$

Low redshift : Doppler formula $\quad z \simeq \frac{v}{c}$

so $\frac{\lambda_{obs}}{\lambda_{emit}} \simeq 1 + \frac{v}{c}$ ← infer

measure

Hubble (1929) : distant galaxies receding from us, recession velocity increases linearly with distance



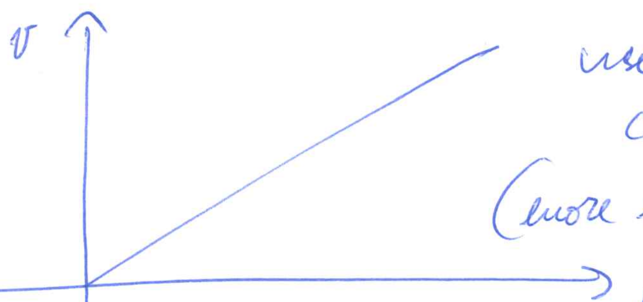
physical distance $d = a\chi$

assume galaxies fixed on the comoving grid, no ___peculiar velocity___ (only receding due to expansion)

$$v = \dot{d} = \dot{(a\chi)} = \dot{a}\chi + a\cancel{\dot{\chi}} =$$

$$= \dot{a}\chi = \dot{a}\frac{d}{a} = \left(\frac{\dot{a}}{a}\right)d = Hd \qquad \text{since } H = \frac{1}{a}\frac{da}{dt}$$

so $v = Hd \propto d$



use standard candles (more later)

Evidence that → Universe is expanding!!!

Beyond $z \ll 1$, $v = Hd$ no longer true

(later you will look at luminosity distance)

Hubble diagram: v vs d or more generally d vs z

z: easy to get, look at spectral lines

d: hard, need objects with intrinsic fixed property (e.g. luminosity)

Type Ia Supernovae (SNeIa) standard candles up to $z \sim 3$

$d(z)$ relation (more later) depends on energy content and cosmological model

Equilibrium and beyond

$\rho_r \propto a^{-4}$ $\rho_m \propto a^{-3}$

Energy densities much higher in the past $\Rightarrow$ hot, dense plasma initially in equilibrium

$\hookrightarrow$ rates for particle interactions also much higher : Γ

$$\Gamma(z) \quad \overset{?}{\lesseqgtr} \quad H(z)$$

$\Gamma_a(z) > H(z) \Longrightarrow$ particle "a" in equilibrium

$\downarrow$ time

$\Gamma_a(z) < H(z) \Rightarrow$ particle "a" falls out of equilibrium or decouples

Early Universe very simple environment: smooth, hot, most of its constituents in equilibrium

As it cools down particles start falling out of equilibrium ("freeze-out"/"decoupling")

$T \gtrsim \frac{\text{MeV}}{k_B}$: no neutral atoms, no bound nuclei

γ immediately ionizes any atom/nucleus

$T \lesssim$ Binding energy $\rightarrow$ lighter elements can form

Nuclear energies ~ MeV $\rightarrow$ light nuclei form $(D, {}^3He, {}^7Li, {}^4He)$

Big Bang Nucleosynthesis (BBN) $\leftarrow$ $(t \sim 3\,\text{min})$ $z \sim 10^{10}$

Can easily compute abundances of light elements just by knowing early Universe conditions + nuclear cross-sections

Light element abundances (especially D) depend on density of visible matter (protons, neutrons, electrons $\rightarrow$ baryons!) $\rho_b \propto a^{-3}$

$\downarrow$

need non-baryonic dark matter! $\Leftarrow$ $\rho_{b,0} \sim 5\%\ \rho_{crit}$

CMB: Cosmic Microwave Background

$T \sim$ eV, $z \sim 1100$ $(t \sim 300000$ yrs$)$

$\Gamma_{e\gamma \to e\gamma}(z) < H(z)$ $\rightarrow$ photons can start propagating freely through space

$\uparrow$ Compton scattering

$\downarrow$

CMB: earliest "picture" of the Universe!

HUGE amount of information about the early Universe $\leftarrow$

CMB photons have blackbody spectrum

Specific intensity

$$\rightarrow I_\nu = \frac{4\pi\hbar\nu^3/c^2}{\exp\left[2\pi\hbar\nu/k_B T\right]-1} \qquad \left[\frac{\text{energy}}{\text{Area}\cdot\text{time}\cdot\text{solid angle}\cdot\text{frequency}}\right]$$

CMB temperature $T \sim 3K$

CMB <u>almost</u> perfect $T \sim 3K$ blackbody

Tiny ($\sim 10^{-5}$) anisotropies in the sky encode crucial information

$\downarrow$

inhomogeneities $\xrightarrow{\text{gravity}}$ structure

Dark matter, dark energy, and inflation

Hot Big Bang model pillars:

- Hubble~~'s law~~ diagram
- BBN
- CMB

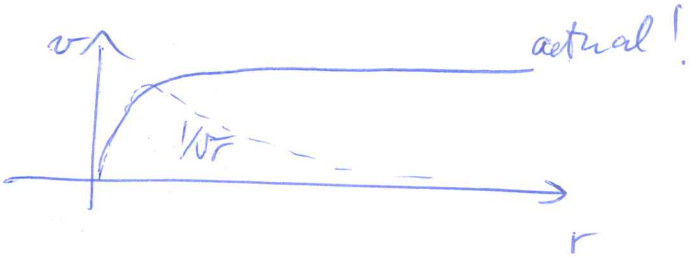
But <u>visible matter</u> ("baryons") cannot be the end of the story

Need some non-luminous <u>dark matter</u>

- BBN
- CMB
- rotation curves

Rotation curves of galaxies

$$\frac{1}{2}mv^2 = \frac{GmM}{r} \longrightarrow v \propto \frac{1}{\sqrt{r}}$$



What is DM?

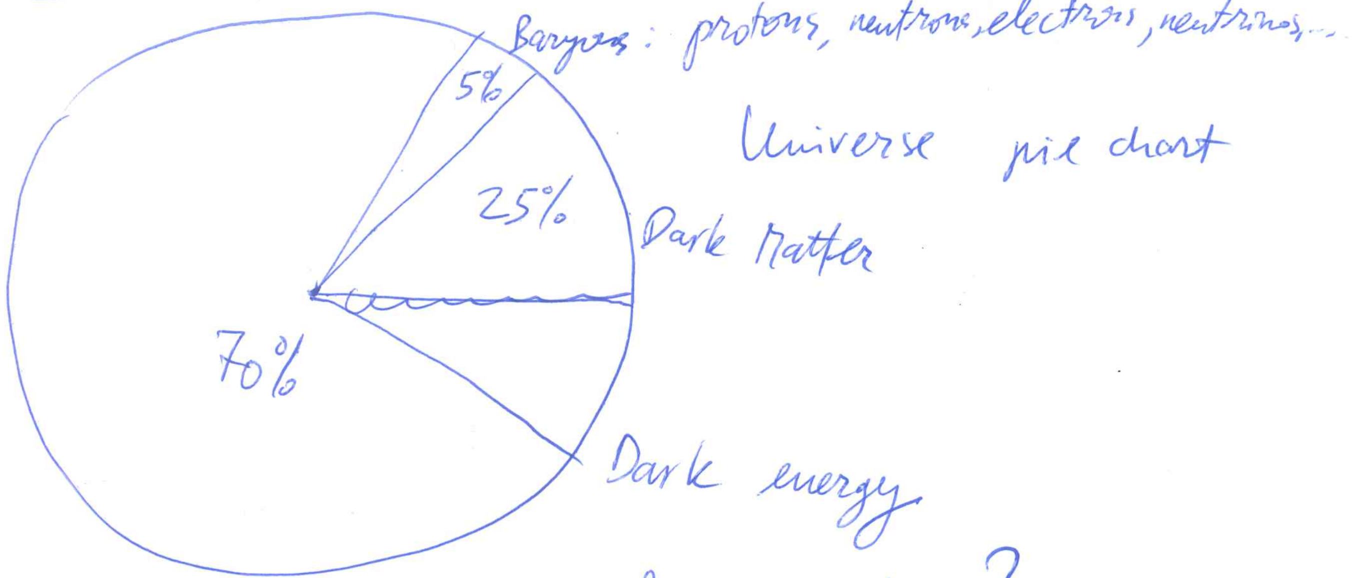
We don't know, but could be new elementary particle which once in equilibrium, then decoupled when $T \sim$ GeV

Also need dark energy to explain why current expansion is accelerated (we will see with just radiation and/or visible or dark matter, expansion is decelerated)

\# We see lots of structure today: grew under gravitational instability starting from tiny seed perturbations which we see as anisotropies in the CMB

~~State~~ who set the seeds for these perturbations?

Inflation! (also solves several other problems) $t \sim 10^{-35}$ s



How to learn about all this from observations?

(mostly) in good agreement with observations

CMB

C_ℓ

ΛCDM model:

Baryons + DM + Λ (DE)

\+ inflation for seed perturbations

\+ GR

LSS] observations

compare

P(k)] $\boxed{\text{Theory}}$ (perturbations)

What this course is about (mostly)