The trouble with spatial curvature

Sunny Vagnozzi

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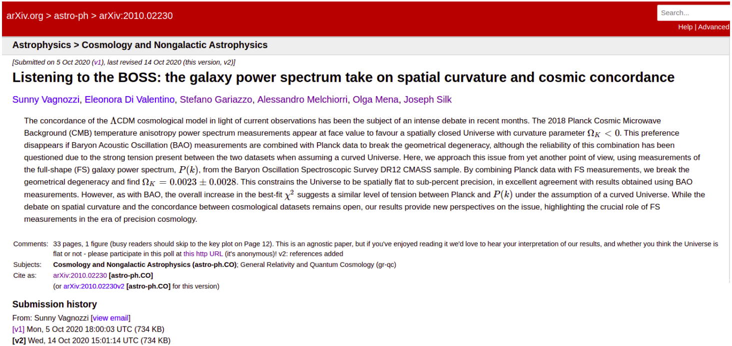
Based on `arXiv:2010.02230`



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Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 5 Oct 2020 (v1), last revised 14 Oct 2020 (this version, v2)]

Listening to the BOSS: the galaxy power spectrum take on spatial curvature and cosmic concordance

Sunny Vagnozzi, Eleonora Di Valentino, Stefano Gariazzo, Alessandro Melchiorri, Olga Mena, Joseph Silk

The concordance of the ΛCDM cosmological model in light of current observations has been the subject of an intense debate in recent months. The 2018 Planck Cosmic Microwave Background (CMB) temperature anisotropy power spectrum measurements appear at face value to favour a spatially closed Universe with curvature parameter $\Omega_K < 0$. This preference disappears if Baryon Acoustic Oscillation (BAO) measurements are combined with Planck data to break the geometrical degeneracy, although the reliability of this combination has been questioned due to the strong tension present between the two datasets when assuming a curved Universe. Here, we approach this issue from yet another point of view, using measurements of the full-shape (FS) galaxy power spectrum, $P(k)$, from the Baryon Oscillation Spectroscopic Survey DR12 CMASS sample. By combining Planck data with FS measurements, we break the geometrical degeneracy and find $\Omega_K = 0.0023 \pm 0.0028$. This constrains the Universe to be spatially flat to sub-percent precision, in excellent agreement with results obtained using BAO measurements. However, as with BAO, the overall increase in the best-fit χ^2 suggests a similar level of tension between Planck and $P(k)$ under the assumption of a curved Universe. While the debate on spatial curvature and the concordance between cosmological datasets remains open, our results provide new perspectives on the issue, highlighting the crucial role of FS measurements in the era of precision cosmology.

Comments: 33 pages, 1 figure (busy readers should skip to the key plot on Page 12). This is an agnostic paper, but if you've enjoyed reading it we'd love to hear your interpretation of our results, and whether you think the Universe is flat or not - please participate in this poll at this http URL (it's anonymous)! v2: references added

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc)

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Eleonora Di Valentino
(Manchester->Durham)

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Alessandro Melchiorri
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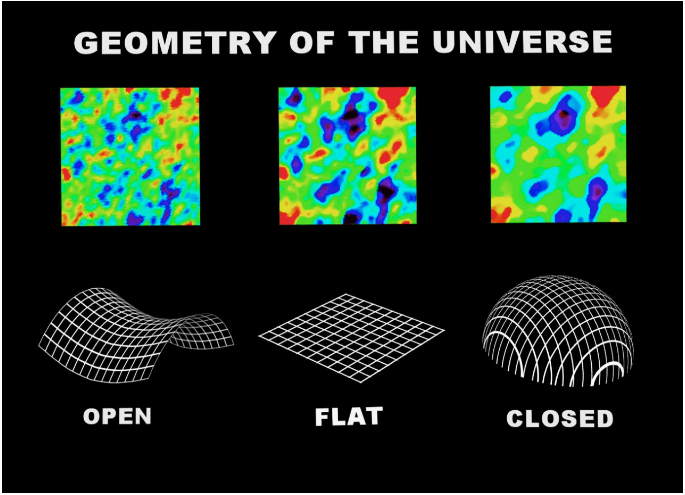
Olga Mena
(Valencia)

Joe Silk
(Oxford/IAP/Johns Hopkins)

What is the shape of the Universe?

What is the ~~shape~~ local geometry of the observable Universe?



Credits: NASA/GSFC

The curvature parameter

$$\begin{aligned} H^2 &= \frac{8\pi G(\rho_m + \rho_\gamma + \rho_\nu)}{3} + \frac{\Lambda}{3} - \frac{k}{R_0^2 a^2} \\ &\downarrow \\ H^2 &= H_0^2 \sqrt{\Omega_m a^{-3} + \Omega_\gamma a^{-4} + \Omega_\nu(a) + \Omega_\Lambda + \Omega_K a^{-2}} \\ \Omega_K &\equiv -\frac{k}{H_0^2 R_0^2} \end{aligned}$$

Ω_K and k come (confusingly) with opposite signs:

$$k = \begin{cases} -1 & \text{spatially open Universe} \quad \Omega_K > 0 \\ +1 & \text{spatially closed Universe} \quad \Omega_K < 0 \\ 0 & \text{spatially flat Universe} \quad \Omega_K = 0 \end{cases}$$

The geometrical degeneracy

Key angular scale:

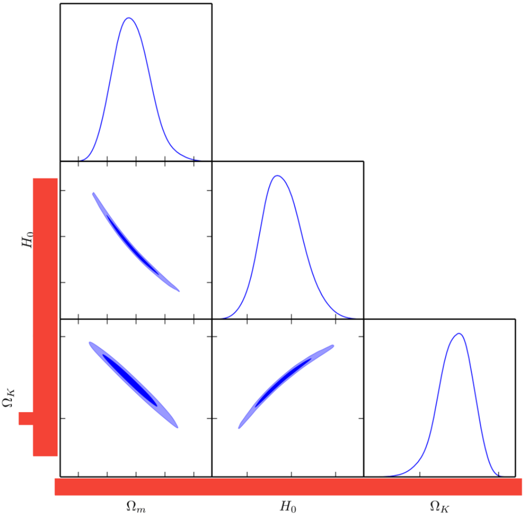
$$\theta_s = \frac{r_s}{D_A} = \frac{\int_z^\infty \frac{dz'}{H(z')}}{\int_0^z \frac{dz''}{H(z'')}}$$

Geometrical degeneracy notably affects Ω_K, H_0, and Ω_m (equivalently Ω_Λ)

Is the Universe:

- young (high H_0) with a large amount of vacuum energy and negative spatial curvature?
- spatially flat?
- old (low H_0) with little vacuum energy and positive spatial curvature?
- ...

The geometrical degeneracy



Credits: *Planck* public chains

How to break the geometrical degeneracy?

Need to pin down post-recombination expansion rate: Ω_m, H_0, $H(z)$,...

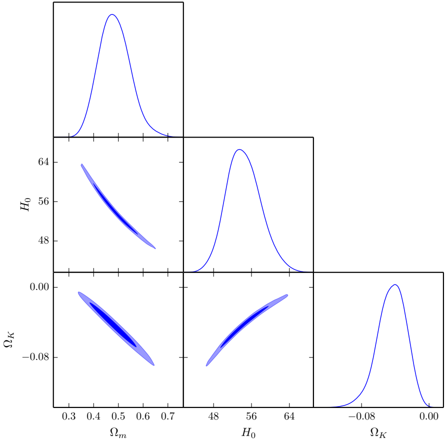
$$D_A(z) = \int_0^z \frac{dz'}{H(z')} \simeq \int_0^z \frac{dz'}{H_0\sqrt{\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + (1-\Omega_m-\Omega_K)}}$$

Examples:

- BAO (D_V/r_s, D_A/r_s, Hr_s → help stabilizing Ω_m and H_0)
- CMB lensing (helps stabilizing Ω_m)
- Uncalibrated SNeIa (*Pantheon*, help stabilizing Ω_m)
- Local Cepheid- or TRGB-calibrated SNeIa measurements of H_0
- This talk: full-shape (FS) galaxy power spectrum
- $++$

Planck 2018 results

Planck TTTEEE+lowE: $\Omega_K = -0.044^{+0.018}_{-0.015}$ → apparent detection of $\Omega_K \neq 0$ at the $\mathcal{O}(10^{-2} - 10^{-1})$ level?



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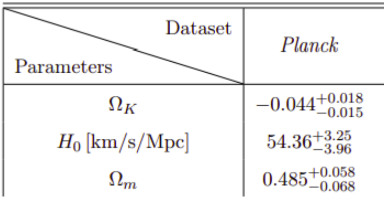
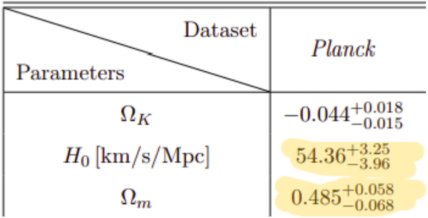


Dataset / Parameters	*Planck*
Ω_K	$-0.044^{+0.018}_{-0.015}$
H_0 [km/s/Mpc]	$54.36^{+3.25}_{-3.96}$
Ω_m	$0.485^{+0.058}_{-0.068}$

Planck 2018 results

Rather implausible (to say the least) values of H_0 and Ω_m within ΛCDM+Ω_K 7-parameter model ($K\Lambda$CDM)



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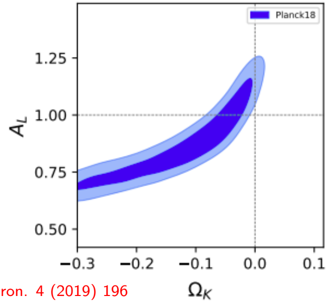
Ω_m	$0.485^{+0.058}_{-0.068}$

H_0 in strong tension with whatever local measurement you can think about (Cepheid- and TRGB-calibrated SNeIa, megamasers, H0LiCOW strong lensing,...)

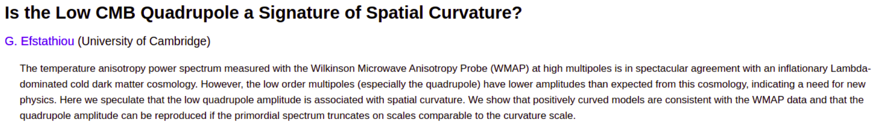
Ω_m also in strong tension with late-time measurements (cosmic shear, cluster counts,...)

Where does this come from?

Partly (but not entirely) from the lensing/A_{lens} anomaly



Credits: Di Valentino *et al.*, Nat. Astron. 4 (2019) 196



Is the Low CMB Quadrupole a Signature of Spatial Curvature?

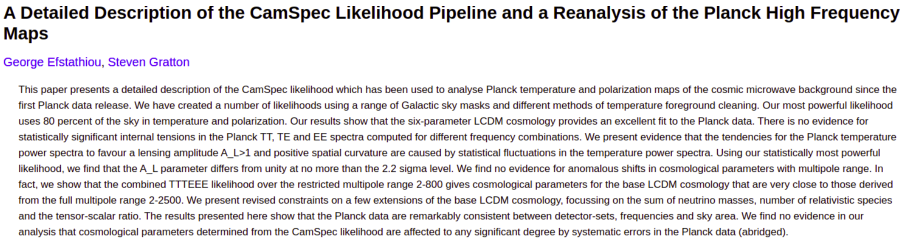
G. Efstathiou (University of Cambridge)

The temperature anisotropy power spectrum measured with the Wilkinson Microwave Anisotropy Probe (WMAP) at high multipoles is in spectacular agreement with an inflationary Lambda-dominated cold dark matter cosmology. However, the low order multipoles (especially the quadrupole) have lower amplitudes than expected from this cosmology, indicating a need for new physics. Here we speculate that the low quadrupole amplitude is associated with spatial curvature. We show that positively curved models are consistent with the WMAP data and that the quadrupole amplitude can be reproduced if the primordial spectrum truncates on scales comparable to the curvature scale.

Efstathiou, MNRAS 343 (2003) L95

Is this a fluke?

Significance of anomalies appears to decrease with more data (=access to higher sky fraction - using `12.5HMCl CamSpec` likelihood)...



A Detailed Description of the CamSpec Likelihood Pipeline and a Reanalysis of the Planck High Frequency Maps

George Efstathiou, Steven Gratton

This paper presents a detailed description of the CamSpec likelihood which has been used to analyse Planck temperature and polarization maps of the cosmic microwave background since the first Planck data release. We have created a number of likelihoods using a range of Galactic sky masks and different methods of temperature foreground cleaning. Our most powerful likelihood uses 80 percent of the sky in temperature and polarization. Our results show that the six-parameter LCDM cosmology provides an excellent fit to the Planck data. There is no evidence for statistically significant internal tensions in the Planck TT, TE and EE spectra computed for different frequency combinations. We present evidence that the tendencies for the Planck temperature power spectra to favour a lensing amplitude A_L>1 and positive spatial curvature are caused by statistical fluctuations in the temperature power spectra. Using our statistically most powerful likelihood, we find that the A_L parameter differs from unity at no more than the 2.2 sigma level. We find no evidence for anomalous shifts in cosmological parameters with multipole range. In fact, we show that the combined TTTEEE likelihood over the restricted multipole range 2-800 gives cosmological parameters for the base LCDM cosmology that are very close to those derived from the full multipole range 2-2500. We present revised constraints on a few extensions of the base LCDM cosmology, focussing on the sum of neutrino masses, number of relativistic species and the tensor-scalar ratio. The results presented here show that the Planck data are remarkably consistent between detector-sets, frequencies and sky area. We find no evidence in our analysis that cosmological parameters determined from the CamSpec likelihood are affected to any significant degree by systematic errors in the Planck data (abridged).

Efstathiou & Gratton, arXiv:1910.00483

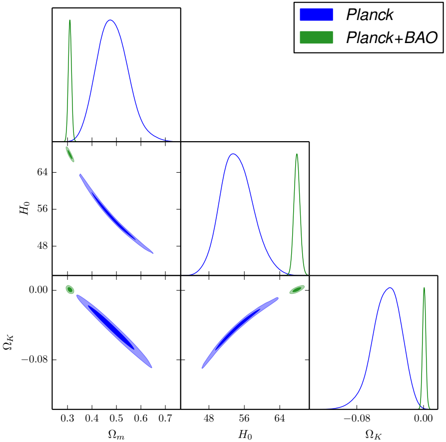
...as one would expect if this were a fluke!

ACT DR4 (+WMAP) results consistent with $A_L = 1$ and $\Omega_K = 0$, no sign of lensing anomaly, support fluke interpretation Aiola *et al.*, arXiv:2007.07288

Breaking the geometrical degeneracy

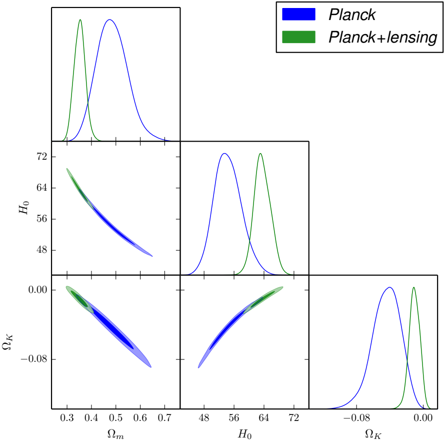
Examples: *Planck* TTTEEE+lowl+lowE

+BAO



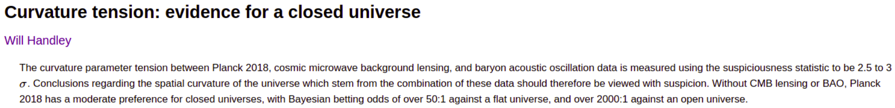
Credits: *Planck* public chains

+CMB lensing



Credits: *Planck* public chains

Lots of subsequent discussion

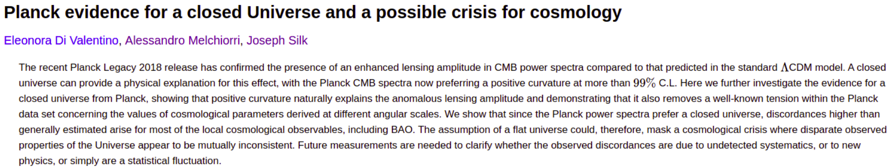


Curvature tension: evidence for a closed universe

Will Handley

The curvature parameter tension between Planck 2018, cosmic microwave background lensing, and baryon acoustic oscillation data is measured using the suspiciousness statistic to be 2.5 to 3 σ. Conclusions regarding the spatial curvature of the universe which stem from the combination of these data should therefore be viewed with suspicion. Without CMB lensing or BAO, Planck 2018 has a moderate preference for closed universes, with Bayesian betting odds of over 50:1 against a flat universe, and over 2000:1 against an open universe.

Handley, arXiv:1908.09139

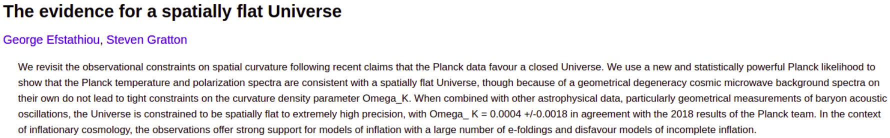


Planck evidence for a closed Universe and a possible crisis for cosmology

Eleonora Di Valentino, Alessandro Melchiorri, Joseph Silk

The recent Planck Legacy 2018 release has confirmed the presence of an enhanced lensing amplitude in CMB power spectra compared to that predicted in the standard ΛCDM model. A closed universe can provide a physical explanation for this effect, with the Planck CMB spectra now preferring a positive curvature at more than 99% C.L. Here we further investigate the evidence for a closed universe from Planck, showing that positive curvature naturally explains the anomalous lensing amplitude and demonstrating that it also removes a well-known tension within the Planck data set concerning the values of cosmological parameters derived at different angular scales. We show that since the Planck power spectra prefer a closed universe, discordances higher than generally estimated arise for most of the local cosmological observables, including BAO. The assumption of a flat universe could, therefore, mask a cosmological crisis where disparate observed properties of the Universe appear to be mutually inconsistent. Future measurements are needed to clarify whether the observed discordances are due to undetected systematics, or to new physics, or simply are a statistical fluctuation.

Di Valentino *et al.*, Nat. Astron. 4 (2019) 196



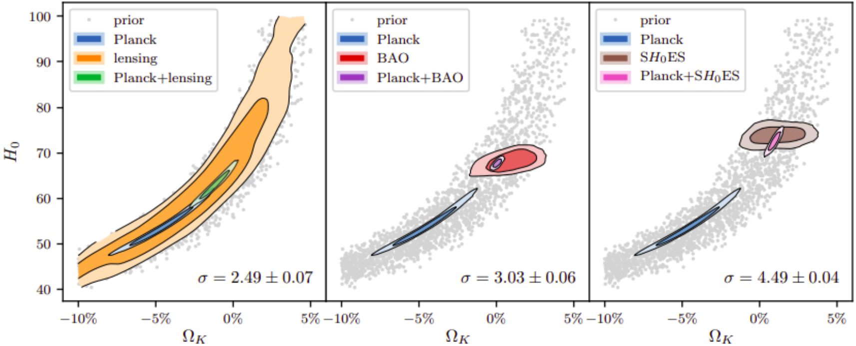
The evidence for a spatially flat Universe

George Efstathiou, Steven Gratton

We revisit the observational constraints on spatial curvature following recent claims that the Planck data favour a closed Universe. We use a new and statistically powerful Planck likelihood to show that the Planck temperature and polarization spectra are consistent with a spatially flat Universe, though because of a geometrical degeneracy cosmic microwave background spectra on their own do not lead to tight constraints on the curvature density parameter Omega_K. When combined with other astrophysical data, particularly geometrical measurements of baryon acoustic oscillations, the Universe is constrained to be spatially flat to extremely high precision, with Omega_ K = 0.0004 +/-0.0018 in agreement with the 2018 results of the Planck team. In the context of inflationary cosmology, the observations offer strong support for models of inflation with a large number of e-foldings and disfavour models of incomplete inflation.

Efstathiou & Gratton, MNRAS 496 (2020) L91

Tensions with external datasets?

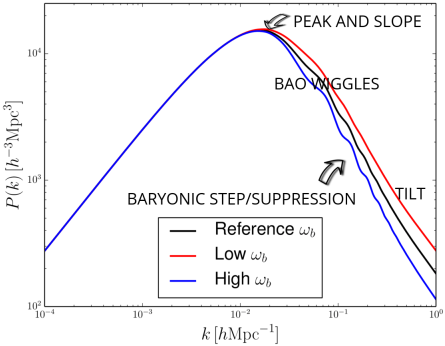


Credits: Handley, arXiv:1908.09139

Should we believe results coming from the combination of datasets in tension *within a given model*?
Can we break the geometrical degeneracy in a different way?

How can FS break the geometrical degeneracy?

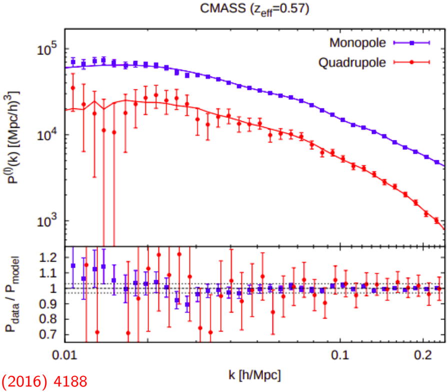
Plot made keeping Ω_m and $z_{\rm eq}$ fixed by adjusting H_0



- Position of BAO wiggles in k space $\rightarrow D_V \rightarrow H_0$
- $k_{\rm eq}$ turnaround in $P(k) \rightarrow$ shape parameter $\Gamma \equiv \Omega_m h$
- Baryonic step/suppression $\rightarrow \Omega_b h^2$ (hard to measure)

FS data

Monopole of BOSS DR12 CMASS power spectrum measured by Gil-Marín *et al.* [1] (conservative $k_{\mathrm{max}} = 0.135\, h\, \mathrm{Mpc}^{-1}$ cutoff)



Gil-Marín *et al.*, MNRAS 460 (2016) 4188

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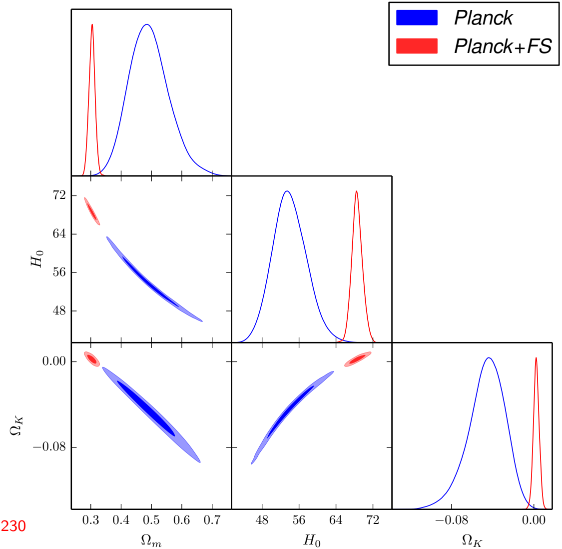
[1] Note: 1) not the same $P(k)$ quoted in "consensus" BOSS results (but gives consistent results); 2) not the same $P(k)$ used by recent EFTofLSS analyses

Combining *Planck* and FS data

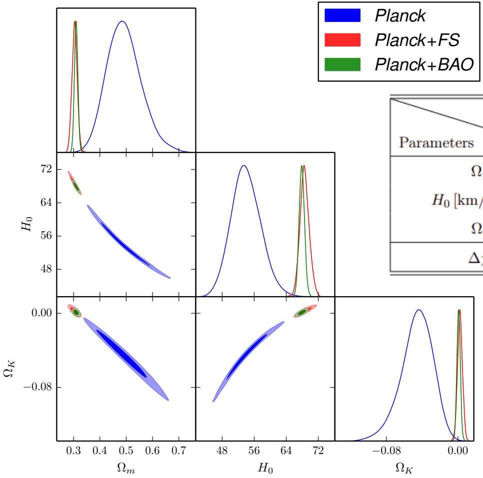
Planck+*FS*: $\Omega_K = 0.0023 \pm 0.0028 \rightarrow$ consistent with $\Omega_K = 0$ @$< 1\sigma$



SV *et al.*, arXiv:2010.02230

Compare FS and BAO

1. Consistent results across the two → good sanity check!
2. Sensible values for H_0 and Ω_m (also a good sanity check)
3. Much smaller $\Delta\chi^2$ (additional Ω_k parameter not preferred)

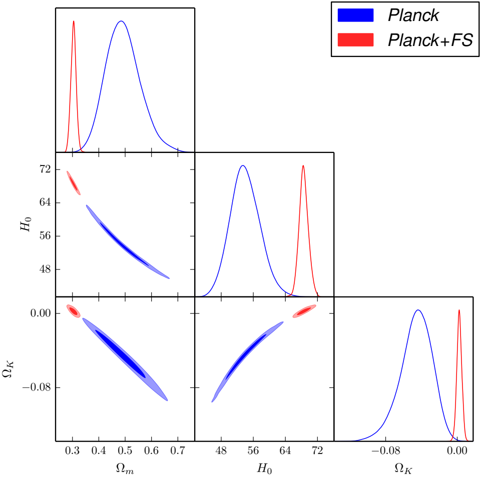


Dataset Parameters	*Planck*	*Planck+BAO*	*Planck+FS*
Ω_K	$-0.044^{+0.018}_{-0.015}$	0.0008 ± 0.0019	0.0023 ± 0.0028
H_0 [km/s/Mpc]	$54.36^{+3.25}_{-3.96}$	67.88 ± 0.66	$68.59^{+1.08}_{-1.20}$
Ω_m	$0.485^{+0.058}_{-0.068}$	0.310 ± 0.007	0.304 ± 0.010
$\Delta\chi^2$	-10.9	-0.6	-1.0

SV *et al.*, arXiv:2010.02230

Tensions between *Planck* and FS?

We all see a 3σish tension by eye...



SV *et al.*, arXiv:2010.02230

Tensions between *Planck* and FS?

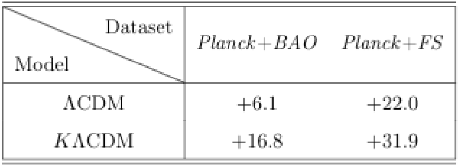
We use a deviance information criterion (DIC)-based tension metric:

Used in KiDS and CFHTLenS, see e.g. Hildebrandt *et al.*, MNRAS 465 (2017) 1454; Joudaki *et al.*

$$\mathcal{I}(D_1\,,D_2)=\exp\left[-\frac{\mathrm{DIC}(D_1\cup D_2)-\mathrm{DIC}(D_1)-\mathrm{DIC}(D_2)}{2}\right]$$

If datasets concordant then $\log\mathcal{I}>0$, else $\log\mathcal{I}<0$ if discordant

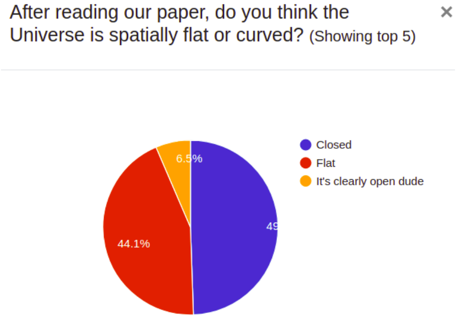
With D_1 =*Planck*, D_2=*FS* $\rightarrow \log\mathcal{I}\approx -2.5$
strong tension on a Jeffreys-like scale → see this by eye (and from χ^2s)



Dataset Model	*Planck+BAO*	*Planck+FS*
ΛCDM	+6.1	+22.0
*K*ΛCDM	+16.8	+31.9

An impasse?

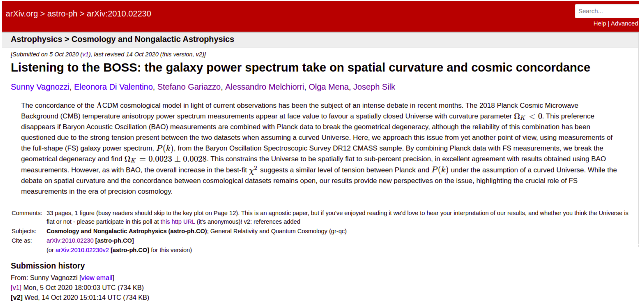
- We want to break the geometrical degeneracy with external datasets ("ext") to stabilize *Planck* constraints on Ω_K...
- ...but always run into tensions when doing so within $K\Lambda$CDM...
- ...including when using FS to break the geometrical degeneracy!
- *Planck*+ext always points towards $\Omega_K = 0$, including "ext"=FS
- Q: can we believe these results/dataset combinations?
  (I'll be agnostic and not answer this one)



After reading our paper, do you think the Universe is spatially flat or curved? (Showing top 5)

Conclusions

- Used FS galaxy power spectrum to break the geometrical degeneracy
- *Planck*+*FS* gives $\Omega_K = 0.0023 \pm 0.0028$ (good news for inflation)...
- ...but *Planck* and *FS* are in tension within ΛCDM+Ω_K!
- Where to from here: check stability against extended parameter space, use better tension measures (*e.g.* suspiciousness), push to higher $k_{\rm max}$ with better theoretical modelling (*e.g.* EFTofLSS),...
- Read more in `arXiv:2010.02230`!



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Astrophysics > Cosmology and Nongalactic Astrophysics

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Subjects:	**Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc)
Cite as:	arXiv:2010.02230 **[astro-ph.CO]** (or arXiv:2010.02230v2 **[astro-ph.CO]** for this version)

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[v1] Mon, 5 Oct 2020 18:00:03 UTC (734 KB)
[v2] Wed, 14 Oct 2020 15:01:14 UTC (734 KB)

Backup slides

FS theoretical modelling

Alcock-Paczynski effect, RSD, Fingers-of-God, galaxy bias, shot noise:

$$\begin{aligned} P_g^{\rm th}(k, z_{\rm eff}) &= \frac{D^2_{A,\rm fid}(z_{\rm eff})}{D^2_A(z_{\rm eff})} \frac{H(z_{\rm eff})}{H_{\rm fid}(z_{\rm eff})} \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right) \exp\left[-\left(\hat{k}\sigma_{\rm FoG}\right)^2\right] \\ &\times\ b^2(\hat{k}) P_{m,\rm HF}(\hat{k}, z_{\rm eff}) + P_s \end{aligned}$$

where:

$$\begin{aligned} \hat{k} &= k \left[\frac{D^2_A(z_{\rm eff})}{D^2_{A,\rm fid}(z_{\rm eff})} \frac{H_{\rm fid}(z_{\rm eff})}{H(z_{\rm eff})}\right]^{\frac{1}{3}} \\ \beta(\hat{k}, z_{\rm eff}) &= \frac{f(\hat{k}, z_{\rm eff})}{b_0} = \frac{1}{b_0} \frac{d \ln \sqrt{P_m(\hat{k}, z_{\rm eff})}}{da} \\ f(\hat{k}, z_{\rm eff}) &\approx \Omega_m(z_{\rm eff})^{0.545} = \frac{H_0^2}{H^2(z_{\rm eff})} \Omega_{m,0}(1 + z_{\rm eff})^3 \\ b(\hat{k}) &= b_1 + b_2 \hat{k}^2 \end{aligned}$$

See also modelling for Euclid $P(k)$ forecasts in Sprenger *et al.*, JCAP 1902 (2019) 047

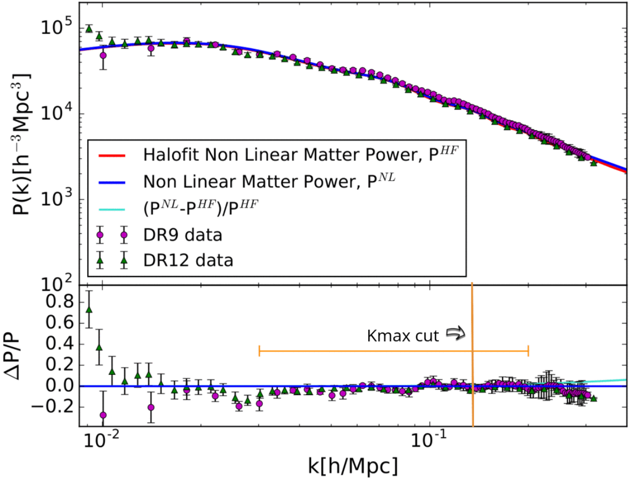
FS observational modelling

Corrections for observational effects (window function) and systematics:

$$
\begin{aligned}
P_g^{\rm conv}(k_i) &= \sum_{ij} W_{ij} P_g^{\rm th}(k_j) - \frac{\sum_j W_{0j} P_g^{\rm th}(k_j)}{P_w(0)} P_w(k_i)\,, \\
P_g^{\rm sys}(k) &= P_g^{\rm conv}(k) + S\left[P_g^{\rm meas}(k) - P_g^{\rm nosys}(k)\right] \\
\ln \mathcal{L}_{FS} &= -\frac{\Delta^T C^{-1} \Delta}{2}\,, \quad \Delta \equiv P_g^{\rm meas} - P_g^{\rm sys}
\end{aligned}
$$

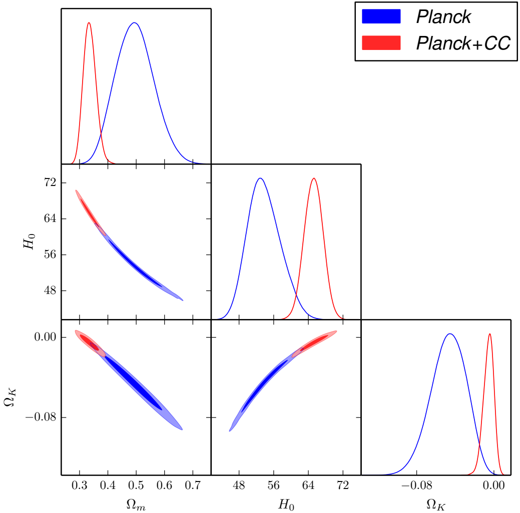
Follows Ross *et al.*, MNRAS 428 (2013) 1116; Beutler *et al.*, MNRAS 424 (2014) 564

Comparison to emulators

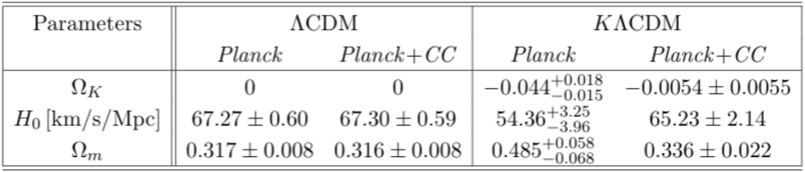


SV *et al.*, PRD 96 (2017) 123503

Cosmic chronometers to break the geometrical degeneracy



Cosmic chronometers to break the geometrical degeneracy



Parameters	ΛCDM		$K\Lambda$CDM	
	Planck	*Planck+CC*	*Planck*	*Planck+CC*
Ω_K	0	0	$-0.044^{+0.018}_{-0.015}$	-0.0054 ± 0.0055
H_0 [km/s/Mpc]	67.27 ± 0.60	67.30 ± 0.59	$54.36^{+3.25}_{-3.96}$	65.23 ± 2.14
Ω_m	0.317 ± 0.008	0.316 ± 0.008	$0.485^{+0.058}_{-0.068}$	0.336 ± 0.022

Cosmic chronometers to break the geometrical degeneracy