## The trouble with spatial curvature

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UNIVERSITY OF CAMBRIDGE

What is the shape of the Universe?

> What is the shape of the Universe? What is the sign of the spatial curvature parameter $\Omega_{K} ?$

## What is the shape of the Universe?

## What is the shape of the Universe? What is the sign of the spatial curvature parameter $\Omega_{K}$ ?

- It is true that Planck CMB temperature and polarization data appears to prefer a spatially closed Universe $\left(\Omega_{K}<0\right)$
- However, to learn more we must combine Planck data with external datasets to break the geometrical degeneracy in a reliable way...
- ...and doing so teaches us that the Universe is very likely spatially flat to the $\left|\Omega_{K}\right| \sim \mathcal{O}\left(10^{-2}\right)$ level


## Based on arXiv:2010.02230 and arXiv:2011.11645

Listening to the BOSS: the galaxy power spectrum take on spatial curvature and cosmic concordance
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Abstract. The concordance of the $A \mathrm{CDM}$ cosmological model in light of current observations has been the subject of an intense debate in recent months. The 2018 Planck Cosmic Microwave Background (CMB) temperature anisotropy power spectrum measurements appear at face value to favour a spatially closed Universe with carvature parameter $\Omega_{K}<0$. This preference disappears if Baryon Acoustic Oscillation (BAO) measurements are combined with Planck data to break the geometrical degeneracy, although the reliability of this comhination has been questioned due to the strong tension present between the two datasets when assuming a curved Universe. Here, we approach this issue from yet another point of view, Oscillation Spectroscopic Survey DR12 CMASS sample. By combining Planck data with FS measurements, we break the geometrical degeneracy and find $\Omega_{N}=0.0023 \pm 0.0028$. This measurements, we break the geometrical degeneracy and find $\Omega_{K}=0.023 \pm 0.0028$. This constrains the Cniverse to be spatially flat to sub-percent precision, in excellent agreenent in the best-fit $\chi^{2}$ suggests a similar level of tension between Planck and $P(k)$ under the assumption of a curved Universe While the debate on spatial curvature and the concordance between cosmological datasets remains open, our results provide new perspectives on the issue, highlighting the crucial role of FS measurements in the era of precision cosmology.

Eppur è platto.
The cosmic chronometer take on spatial curvature and cosmic concordance

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\text { Sunsy Vagnoza } 0,{ }^{1} \text { Ablaham Loeb } 0,{ }^{2} \text { and Mchuas. Monesco } 0^{1,4}
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## ABSTRACT

The quastion of whether Cosmic Microwave Blackground (CMB) temperature and polarixation data from Plonck favor a spatially chosed Universe with curvature parameter $\Omega_{K}<0$ has been the subject of reerent intense discussions. Attempts to break the geometrical degenerary combining Planck data with external datasets suith as Baryon Acoustic Oxillation (BAO) measurempats all point towards a spatially llat Universe, at the cost of significant tensons with Planch, which make the resulting datast combination problematic. Setting this issue would require identifying a dataset which can boeak the grometrical degeneracy while not incurring in these tensions. In this woek we argue that coamic chronometers (CC), measarements of the expansion rate $B(\mathrm{~s})$ from the relative ages of massive early-type pasively evolving galaxies, are the datases we ate after. Furthernoce, CC come with the additional advautage of being virtually free of conmological model assumptions. Combining Planck 2018 CMB temperature asd polarization data with the latast compilation of CC messurements, we break the goometrical degeneracy and find $\left\}_{K}=-0.0054 \pm 0.0055\right.$, consistent with a spatially fist Universe and competitive with the Planek + BAO constraint. After discussing our results in light of the oldest objects in the Univerec, we assess their stability against against minimal parameter space extensions and CC sytenatics, finding them to be stable ngainst both. We find no substantial tension between Planck and CC data within a non-fat Universe, mating the resulting combination reliable. Our results therefore allow us to nesert with contidence that the Unisense is indeed spatially dat to the $\mathcal{O}\left(10^{-2}\right)$ level, a finding which might possilily settle the ongoing spatial curcature dehate, and lends eren more suppost to the already very sucoesuful inflationary paradigm.

Keyaunds: cosmic hackground radiation - cormological parameters - cosmology: observations - distance scale - galaxies: general

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## What is the shape of the Universe?



What is the local geometry of the observable Universe?

## GEOMETRY OF THE UNIVERSE



OPEN


FLAT

CLOSED

## The curvature parameter

$$
\begin{aligned}
H^{2} & =\frac{8 \pi G\left(\rho_{m}+\rho_{\gamma}+\rho_{\nu}\right)}{3}+\frac{\Lambda}{3}-\frac{k}{R_{0}^{2} a^{2}} \\
& \downarrow \\
H^{2} & =H_{0}^{2} \sqrt{\Omega_{m} a^{-3}+\Omega_{\gamma} a^{-4}+\Omega_{\nu}(a)+\Omega_{\Lambda}+\Omega_{K} a^{-2}} \\
\Omega_{K} & \equiv-\frac{k}{H_{0}^{2} R_{0}^{2}}
\end{aligned}
$$

$\Omega_{K}$ and $k$ come (confusingly) with opposite signs:
$k= \begin{cases}-1 & \text { spatially open Universe } \quad \Omega_{K}>0 \\ +1 & \text { spatially closed Universe } \quad \Omega_{K}<0 \\ 0 & \text { spatially flat Universe } \Omega_{K}=0\end{cases}$

## The importance of spatial curvature

Late Universe: sign and value of $\Omega_{K}$ plays a key role in determining the future evolution of the Universe


Early Universe: many inflation models predict (constructed to give) $\Omega_{K} \sim 0$

Measurement of $\left|\Omega_{K}\right| \gtrsim \mathcal{O}\left(10^{-4}\right)$ would be a problem for many inflationary models

Generally easier to accommodate open rather than closed Universe from inflation

## The Planck satellite



Credits: Planck collaboration and ESA

## CMB power spectrum



## Planck 2018 temperature power spectrum



## The geometrical degeneracy



How far away is this person (hopefully more than 2 m ) ? d How tall is this person? $h$
Only data: angle subtended by this person $\theta \approx h / d$
You can't disentangle distance and height from this data alone: geometrical degeneracy!

## Breaking the geometrical degeneracy



Answer: roughly 7 m away and roughly 3 m tall

## The geometrical degeneracy

Key angular scale:

$$
\theta_{s}=\frac{r_{s}\left(z_{\mathrm{LS}}\right)}{D_{A}\left(z_{\mathrm{LS}}\right)}=\frac{\int_{z_{\mathrm{LS}}}^{\infty} \frac{d z^{\prime}}{H\left(z^{\prime}\right)}}{\int_{0}^{z_{\mathrm{LS}}} \frac{d z^{\prime \prime}}{H\left(z^{\prime \prime}\right)}}
$$

Geometrical degeneracy notably affects $\Omega_{K}, H_{0}$, and $\Omega_{m}$ (equivalently $\Omega_{\Lambda}$ )
Is the Universe:

- young (high $H_{0}$ ) with a large amount of vacuum energy and negative spatial curvature?
- spatially flat?
- old (low $H_{0}$ ) with little vacuum energy and positive spatial curvature?
- ...


## The geometrical degeneracy



## How to break the geometrical degeneracy?

Need to pin down post-recombination expansion rate: $\Omega_{m}, H_{0}, H(z), \ldots$
$D_{A}(z)=\int_{0}^{z} \frac{d z^{\prime}}{H\left(z^{\prime}\right)} \simeq \int_{0}^{z} \frac{d z^{\prime}}{H_{0} \sqrt{\Omega_{m}\left(1+z^{\prime}\right)^{3}+\Omega_{K}\left(1+z^{\prime}\right)^{2}+\left(1-\Omega_{m}-\Omega_{K}\right)}}$
Examples:

- BAO $\left(D_{V} / r_{s}, D_{A} / r_{s}, H r_{s} \rightarrow\right.$ help stabilizing $\Omega_{m}$ and $\left.H_{0}\right)$
- CMB lensing (helps stabilizing $\Omega_{m}$ )
- Uncalibrated SNela (Pantheon, help stabilizing $\Omega_{m}$ )
- Local Cepheid- or TRGB-calibrated SNela measurements of $H_{0}$
- ++ (cluster counts, weak lensing, X -ray gas mass fraction,...)
- This talk: full-shape (FS) galaxy power spectrum
- This talk: cosmic chronometers (CC)


## Planck 2018 results

Planck TTTEEE+lowE: $\Omega_{K}=-0.044_{-0.015}^{+0.018} \rightarrow$ apparent detection of $\Omega_{K} \neq 0$ at the $\mathcal{O}\left(10^{-2}-10^{-1}\right)$ level?


| Parameters | Planck |
| :---: | :---: |
| $\Omega_{K}$ | $-0.044_{-0.015}^{+0.018}$ |
| $H_{0}[\mathrm{~km} / \mathrm{s} / \mathrm{Mpc}]$ | $54.36_{-3.96}^{+3.25}$ |
| $\Omega_{m}$ | $0.485_{-0.068}^{+0.058}$ |

[^0]
## Planck 2018 results

Rather implausible (to say the least) values of $H_{0}$ and $\Omega_{m}$ within $\Lambda C D M+\Omega_{K} 7$-parameter model ( $K \wedge C D M$ )

|  | Dataset |
| :---: | :---: |
| Parameters | Planck |
| $\Omega_{K}$ | $-0.044_{-0.015}^{+0.018}$ |
| $H_{0}[\mathrm{~km} / \mathrm{s} / \mathrm{Mpc}]$ | $54.36_{-3.96}^{+3.25}$ |
| $\Omega_{m}$ | $0.485_{-0.068}^{+0.058}$ |

$H_{0}$ in strong tension with whatever local measurement you can think about (Cepheid- and TRGBcalibrated SNela, megamasers, H0LiCOW strong lensing, ...)
$\Omega_{m}$ also in strong tension with late-time measurements (cosmic shear, cluster counts,...)

## Where does this come from?

## Partly (but not entirely) from the lensing $/ A_{\text {lens }}$ anomaly



Is the Low CMB Quadrupole a Signature of Spatial Curvature?
G. Efstathiou (University of Cambridge)

The temperature anisotropy power spectrum measured with the Wilkinson Microwave Anisotropy Probe (WMAP) at high multipoles is in spectacular agreement with an inflationary Lambdadominated cold dark matter cosmology. However, the low order multipoles (especially the quadrupole) have lower amplitudes than expected from this cosmology, indicating a need for new physics. Here we speculate that the low quadrupole amplitude is associated with spatial curvature. We show that positively curved models are consistent with the WMAP data and that the quadrupole amplitude can be reproduced if the primordial spectrum truncates on scales comparable to the curvature scale.

## Is this a fluke?

# Significance of anomalies appears to decrease with more data (=access to higher sky fraction - using 12.5HMCl CamSpec likelihood)... 

## A Detailed Description of the CamSpec Likelihood Pipeline and a Reanalysis of the Planck High Frequency Maps

George Efstathiou, Steven Gratton

This paper presents a detailed description of the CamSpec likelihood which has been used to analyse Planck temperature and polarization maps of the cosmic microwave background since the first Planck data release. We have created a number of likelihoods using a range of Galactic sky masks and different methods of temperature foreground cleaning. Our most powerful likelihood uses 80 percent of the sky in temperature and polarization. Our results show that the six-parameter LCDM cosmology provides an excellent fit to the Planck data. There is no evidence for statistically significant internal tensions in the Planck TT, TE and EE spectra computed for different frequency combinations. We present evidence that the tendencies for the Planck temperature power spectra to favour a lensing amplitude $A \_L>1$ and positive spatial curvature are caused by statistical fluctuations in the temperature power spectra. Using our statistically most powerful likelihood, we find that the A L parameter differs from unity at no more than the 2.2 sigma level. We find no evidence for anomalous shifts in cosmological parameters with multipole range. In fact, we show that the combined TTTEEE likelihood over the restricted multipole range 2-800 gives cosmological parameters for the base LCDM cosmology that are very close to those derived from the full multipole range 2-2500. We present revised constraints on a few extensions of the base LCDM cosmology, focussing on the sum of neutrino masses, number of relativistic species and the tensor-scalar ratio. The results presented here show that the Planck data are remarkably consistent between detector-sets, frequencies and sky area. We find no evidence in our analysis that cosmological parameters determined from the CamSpec likelihood are affected to any significant degree by systematic errors in the Planck data (abridged).

Efstathiou \& Gratton, arXiv:1910.00483
...as one would expect if this were a fluke!

## ACT DR4 (+WMAP) results consistent with $A_{L}=1$ and $\Omega_{K}=0$, no sign of lensing anomaly, support fluke interpretation Aiola et al., arxiv:2007.07288

## Breaking the geometrical degeneracy

## Example: Planck TTTEEE+lowl+lowE+BAO



## Lots of subsequent discussion

## Curvature tension: evidence for a closed universe

Will Handley

The curvature parameter tension between Planck 2018, cosmic microwave background lensing, and baryon acoustic oscillation data is measured using the suspiciousness statistic to be 2.5 to 3
$\sigma$. Conclusions regarding the spatial curvature of the universe which stem from the combination of these data should therefore be viewed with suspicion. Without CMB lensing or BAO, Planck
2018 has a moderate preference for closed universes, with Bayesian betting odds of over 50:1 against a flat universe, and over 2000:1 against an open universe.

```
Handley, arXiv:1908.09139
```


## Planck evidence for a closed Universe and a possible crisis for cosmology

Eleonora Di Valentino, Alessandro Melchiorri, Joseph Silk





 physics, or simply are a statistical fluctuation.
Di Valentino et al., Nat. Astron. 4 (2019) 196

## The evidence for a spatially flat Universe

## George Efstathiou, Steven Gratton

We revisit the observational constraints on spatial curvature following recent claims that the Planck data favour a closed Universe. We use a new and statistically powerful Planck likelihood to show that the Planck temperature and polarization spectra are consistent with a spatially flat Universe, though because of a geometrical degeneracy cosmic microwave background spectra on their own do not lead to tight constraints on the curvature density parameter Omega_K. When combined with other astrophysical data, particularly geometrical measurements of baryon acoustic oscillations, the Universe is constrained to be spatially flat to extremely high precision, with Omega_ $K=0.0004+/-0.0018$ in agreement with the 2018 results of the Planck team. In the context of inflationary cosmology, the observations offer strong support for models of inflation with a large number of e-foldings and disfavour models of incomplete inflation.
Efstathiou \& Gratton, MNRAS 496 (2020) L91

## Lots of media attention

# What Shape Is the Universe? A New Study Suggests We've Got It All Wrong 

- 04 When researchers reanalyzed the gold-standard data set of the early universe, they concluded that the cosmos must be "closed," or curled up
Credits: Quanta Magazine
like $a$ ball. Most others remain unconvinced.


Credits: Scientific American new study suggests the cosmos may be curved in upon itself like a ball-but many experts remain unconvinced

## Tensions with external datasets?



Credits: Handley, arXiv:1908.09139
Should we believe results coming from the combination of datasets in tension within a given model?
Can we break the geometrical degeneracy in a different way?

## Tensions with external datasets

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## Breaking the geometrical degeneracy with full-shape galaxy power spectrum data

arXiv.org > astro-ph > arXiv:2010.02230

## Astrophysics > Cosmology and Nongalactic Astrophysics

## [Submitted on 5 Oct 2020 (v1), last revised 14 Oct 2020 (this version, v2)]

## Listening to the BOSS: the galaxy power spectrum take on spatial curvature and cosmic concordance

Sunny Vagnozzi, Eleonora Di Valentino, Stefano Gariazzo, Alessandro Melchiorri, Olga Mena, Joseph Silk

> The concordance of the $\Lambda \mathrm{CDM}$ cosmological model in light of current observations has been the subject of an intense debate in recent months. The 2018 Planck Cosmic Microwave Background (CMB) temperature anisotropy power spectrum measurements appear at face value to favour a spatially closed Universe with curvature parameter $\Omega_{K}<0$. This preference disappears if Baryon Acoustic Oscillation (BAO) measurements are combined with Planck data to break the geometrical degeneracy, although the reliability of this combination has been
debate on spatial curvature and the concordance between cosmological datasets remains open, our results provide new perspectives on the issue, highlighting the crucial role of FS
measurements in the era of precision cosmology.
 flat or not - please participate in this poll at this http URL (it's anonymous)! v2: references added
Subjects: Cosmology and Nongalactic Astrophysics (astro-ph.CO); General Relativity and Quantum Cosmology (gr-qC)
Cite as: arXiv:2010.02230 [astro-ph.co]
(or arXiv:2010.02230v2 [astro-ph.cO] for this version]

## Submission history

From: Sunny Vagnozzi [view email]
[v1] Mon, 5 Oct 2020 18:00:03 UTC (734 KB)
[v2] Wed, 14 Oct 2020 15:01:14 UTC ( 734 KB )

## How can FS break the geometrical degeneracy?



- Position of BAO wiggles in $k$ space $\rightarrow D_{V} \rightarrow H_{0}$
- $k_{\text {eq }}$ turnaround in $P(k) \rightarrow$ shape parameter $\Gamma \equiv \Omega_{m} h$
- Baryonic step/suppression $\rightarrow \Omega_{b} h^{2}$ (hard to measure)
- The CMB already gives us $\Omega_{m} h^{2} \rightarrow$ disentangle $\Omega_{m}$ and $H_{0}$


## FS data

Monopole of pre-reconstructed BOSS DR12 CMASS power spectrum measured by Gil-Marín et al. ${ }^{1}$ (conservative $k_{\max }=0.135 h \mathrm{Mpc}^{-1}$ cutoff)


Gil-Marín et al., MNRAS 460 (2016) 4188
k [h/Mpc]
${ }^{1}$ Note: 1) not the same $P(k)$ quoted in "consensus" BOSS results (but gives consistent results); 2) not the same $P(k)$ used by recent EFTofLSS analyses

## Combining Planck and FS data

Planck $+F S: \Omega_{K}=0.0023 \pm 0.0028 \rightarrow$ consistent with $\Omega_{K}=0 @<1 \sigma$


## Compare FS and BAO

(1) Consistent results across the two $\rightarrow$ good sanity check!
(2) Sensible values for $H_{0}$ and $\Omega_{m}$ (also a good sanity check)
(3) Much smaller $\Delta \chi^{2}$ (additional $\Omega_{k}$ parameter not preferred)


|  | Dataset | Planck | Planck + BAO |
| :---: | :---: | :---: | :---: |
| Parameters |  |  |  |
| $\Omega_{K}$ | $-0.044_{-0.015}^{+0.018}$ | $0.0008 \pm 0.0019$ | $0.0023 \pm 0.0028$ |
| $H_{0}[\mathrm{~km} / \mathrm{s} / \mathrm{Mpc}]$ | $54.36_{-3.96}^{+3.25}$ | $67.88 \pm 0.66$ | $68.59_{-1.20}^{+1.08}$ |
| $\Omega_{m}$ | $0.485_{-0.068}^{+0.058}$ | $0.310 \pm 0.007$ | $0.304 \pm 0.010$ |
| $\Delta \chi^{2}$ | -10.9 | -0.6 | -1.0 |

SV et al., arXiv:2010.02230

## Tensions between Planck and FS?

We all see a $3 \sigma$ ish tension by eye...


## Tensions between Planck and FS?

We use a deviance information criterion (DIC)-based tension metric:
Used by KiDS and CFHTLenS, see e.g. Hildebrandt et al., MNRAS 465 (2017) 1454; Joudaki et al.

$$
\mathcal{I}\left(D_{1}, D_{2}\right)=\exp \left[-\frac{\operatorname{DIC}\left(D_{1} \cup D_{2}\right)-\operatorname{DIC}\left(D_{1}\right)-\operatorname{DIC}\left(D_{2}\right)}{2}\right]
$$

If datasets concordant then $\log \mathcal{I}>0$, else $\log \mathcal{I}<0$ if discordant

With $D_{1}=$ Planck, $D_{2}=F S \rightarrow \log \mathcal{I} \approx-2.5$ strong tension on a Jeffreys-like scale $\rightarrow$ see this by eye (and from $\chi^{2} s$ )

|  | Dataset | Planck + BAO |
| :---: | :---: | :---: |
| Model | Planck $+F S$ |  |
| $\Lambda \mathrm{CDM}$ | +6.1 | +22.0 |
| $K \Lambda \mathrm{CDM}$ | +16.8 | +31.9 |

## An impasse?

- We want to break the geometrical degeneracy with external datasets ("ext") to stabilize Planck constraints on $\Omega_{K} \ldots$
- ...but always run into tensions when doing so within $K \wedge C D M . .$.
- ...including when using FS to break the geometrical degeneracy!
- Planck+ext always points towards $\Omega_{K}=0$, including "ext" $=F S$
- Another problem: most of these external datasets (e.g. BAO and FS) carry some amount of model-dependence in the form of fiducial cosmological assumptions during data reduction process


## How to exit this impasse?

Need a "golden dataset" which:

- helps to break the geometrical degeneracy once combined with Planck CMB temperature and polarization data
- is not in strong tension with Planck data when working within a non-flat Universe
- is as model-independent as possible


## Cosmic chronometers to the rescue

```
arXiv.org > astro-ph > arXiv:2011.11645
Search...
Help | Advance
Astrophysics > Cosmology and Nongalactic Astrophysics
[Submitted on 23 Nov 2020]
Eppur è piatto? The cosmic chronometer take on spatial curvature and cosmic concordance
Sunny Vagnozzi, Abraham Loeb, Michele Moresco
The question of whether Cosmic Microwave Background (CMB) temperature and polarization data from Planck favor a spatially closed Universe with curvature parameter \Omega}\mp@subsup{\Omega}{K}{}<0\mathrm{ has been the subject of recent intense discussions. Attempts to break the geometrical degeneracy combining Planck data with external datasets such as Baryon Acoustic Oscillation (BAO) measurements all point towards a spatially flat Universe, at the cost of significant tensions with Planck, which make the resulting dataset combination problematic. Settling this issue would require identifying a dataset which can break the geometrical degeneracy while not incurring in these tensions. In this work we argue that cosmic chronometers (CC), measurements of the expansion rate \(H(z)\) from the relative ages of massive early-type passively evolving galaxies, are the dataset we are after. Furthermore, CC come with the additional advantage of being virtually free of cosmological model assumptions. Combining Planck 2018 CMB temperature and polarization data with the latest compilation of CC measurements, we break the geometrical degeneracy and find \(\Omega_{K}=-0.0054 \pm 0.0055\), consistent with a spatially flat Universe and competitive with the Planck+BAO constraint. After discussing our results in light of the oldest objects in the Universe, we assess their stability against against minimal parameter space extensions and CC systematics, finding them to be stable against both. We find no substantial tension between Planck and CC data within a non-flat Universe, making the resulting combination reliable. Our results therefore allow us to assert with confidence that the Universe is indeed spatially flat to the \(\mathcal{O}\left(10^{-2}\right)\) level, a finding which might possibly settle the ongoing spatial curvature debate, and lends even more support to the already very successful inflationary paradigm.
```



``` "flat" in Italian. A "Note added" between conclusions and acknowledgements explains our choice of title
Subjects: Cosmology and Nongalactic Astrophysics (astro-ph.CO); General Relativity and Quantum Cosmology (gr-qc)
Cite as: arXiv:2011.11645 [astro-ph.CO]
(or arxiv:2011.11645v1 [astro-ph.co] for this version)
```


## Submission history

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From: Sunny Vagnozzi [view email]
[v1] Mon, 23 Nov 2020 19:00:01 UTC ( \(5,080 \mathrm{~KB}\) )
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## Cosmic chronometers

Age-redshift relation:

$$
\frac{d t}{d z}=-\frac{1}{(1+z) H(z)}
$$

Take two ensembles of passively evolving galaxies that formed at the same time and are separated by a small redshift interval $\Delta z$ around $z_{\text {eff }}$ :

$$
H\left(z_{\mathrm{eff}}\right)=-\frac{1}{1+z_{\mathrm{eff}}} \frac{\Delta z}{\Delta t}
$$

## THE ASTROPHYSICAL JOURNAL

```
Constraining Cosmological Parameters Based on Relative
Galaxy Ages
Raul Jimenez }\mp@subsup{}{}{1}\mathrm{ and Abraham Loeb}\mp@subsup{}{}{2
O 2002. The American Astronomical Society. All rights reserved. Printed in U.S.A.
The Astrophysical Journal, Volume 573, Number 1

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\section*{Cosmic chronometers}

Use massive, early-time, passively-evolving galaxies (evolving on a much longer timescale than their age differences)


\section*{Advantages with respect to distance measurements}

Luminosity/angular diameter distance:
\[
D_{L}=(1+z) \int_{0}^{z} \frac{d z^{\prime}}{H\left(z^{\prime}\right)} \quad D_{A}=\frac{1}{1+z} \int_{0}^{z} \frac{d z^{\prime}}{H\left(z^{\prime}\right)}
\]

Distances suffer from integral sensitivity to expansion history and parameters such as the dark energy equation of state

CMB acoustic scale:
\[
\theta_{s}=\frac{r_{s}\left(z_{\mathrm{LS}}\right)}{D_{A}\left(z_{\mathrm{LS}}\right)}=\frac{\int_{\mathrm{z}_{\mathrm{LS}}}^{\infty} \frac{d z^{\prime}}{H\left(z^{\prime}\right)}}{\int_{0}^{z_{\mathrm{LS}}} \frac{d z^{\prime \prime}}{H\left(z^{\prime \prime}\right)}}
\]

About half of the contribution to \(D_{A}\left(z_{\mathrm{LS}}\right)\) comes from \(H(z)\) at \(0<z \lesssim 2\)

\section*{Cosmic chronometer measurements}

Sweeping a lot of dust under the carpet, we'll assume these measurements are trustworthy see lots of works in the last 10 years, especially by Michele Moresco


Compiled across the last 20 years in: Jiménez et al., ApJ 593 (2003) 622; Simon et al., PRD 71 (2005) 123001; Stern et al., JCAP 1002 (2010) 008; Moresco et al., JCAP 1207 (2012) 053; Zhang et al., Res. Astron. Astrophy. 14 (2014) 1221; Moresco, MNRAS 450 (2015) L16; Moresco et al., JCAP 1605 (2016) 014; Ratsimbazafy et al., MNRAS 467 (2017) 3239

\section*{Combining Planck and CC data}

Planck \(+C C: \Omega_{K}=-0.0054 \pm 0.0055 \rightarrow\) consistent with \(\Omega_{K}=0 @<1 \sigma\)


\section*{Tensions between Planck and CC?}
\(\log \mathcal{I} \approx-0.43 \rightarrow\) no strong tension \(\rightarrow\) we may trust the Planck + CC dataset combination even within a non-flat Universe!


\section*{Compare Planck+CC to Planck+BAO/FS}

By eye much less tension, yet results still go towards \(\Omega_{K}=0\)


SV et al., arXiv:2010.02230


SV et al., arXiv:2011.11645

\section*{Are cosmic chronometers our "golden dataset"?}

Golden dataset characteristics:
- helps to break the geometrical degeneracy once combined with Planck CMB temperature and polarization data
- is not in strong tension with Planck data when working within a non-flat Universe
- is as model-independent as possible

\section*{Are these results stable against an enlarged parameter space?}

Yes (at least when varying \(w\) or \(M_{\nu}\) )!


\section*{How much are these results affected by CC systematics?}

Very little \((\lesssim 10 \%)\) ! See Moresco et al., ApJ 898 (2020) 82 for systematics study


\section*{Conclusions}
- Curvature parameter \(\Omega_{K}\) is a key quantity in cosmology
- Planck CMB temperature and polarization data prefers \(\Omega_{K}<0 \ldots\)
- but for a reliable result need to break geometrical degeneracy!
- Attempts to break the geometrical degeneracy incur in tensions...
- ...example: Planck+full-shape galaxy power spectrum data \(\rightarrow \Omega_{K}=0.0023 \pm 0.0028\) at the cost of a \(\sim 3 \sigma\) tension
- Cosmic chronometer data can break the geometrical degeneracy without incurring in strong tensions \(\rightarrow \Omega_{K}=-0.0054 \pm 0.0055\)
- Universe is spatially flat to the \(\mathcal{O}\left(10^{-2}\right)\) level

Backup slides

\section*{FS theoretical modelling}

Alcock-Paczynski effect, RSD, Fingers-of-God, galaxy bias, shot noise:
\[
\begin{aligned}
P_{g}^{\mathrm{th}}\left(k, z_{\mathrm{eff}}\right) & =\frac{D_{A, \mathrm{fi}}^{2}\left(z_{\mathrm{eff}}\right)}{D_{A}^{2}\left(z_{\mathrm{eff}}\right)} \frac{H\left(z_{\mathrm{eff}}\right)}{H_{\mathrm{fid}}\left(z_{\mathrm{eff}}\right)}\left(1+\frac{2}{3} \beta+\frac{1}{5} \beta^{2}\right) \exp \left[-\left(\hat{k} \sigma_{\mathrm{FoG}}\right)^{2}\right] \\
& \times b^{2}(\hat{k}) P_{m, \mathrm{HF}}\left(\hat{k}, z_{\mathrm{eff}}\right)+P_{s}
\end{aligned}
\]
where:
\[
\begin{aligned}
\hat{k} & =k\left[\frac{D_{A}^{2}\left(z_{\mathrm{eff}}\right)}{D_{A, \mathrm{fid}}^{2}\left(z_{\mathrm{eff}}\right)} \frac{H_{\mathrm{fid}}\left(z_{\mathrm{eff}}\right)}{H\left(z_{\mathrm{eff}}\right)}\right]^{\frac{1}{3}} \\
\beta\left(\hat{k}, z_{\mathrm{eff}}\right) & =\frac{f\left(\hat{k}, z_{\mathrm{eff}}\right)}{b_{0}}=\frac{1}{b_{0}} \frac{d \ln \sqrt{P_{m}\left(\hat{k}, z_{\mathrm{eff}}\right)}}{d a} \\
f\left(\hat{k}, z_{\mathrm{eff}}\right) & \approx \Omega_{m}\left(z_{\mathrm{eff}}\right)^{0.545}=\frac{H_{0}^{2}}{H^{2}\left(z_{\mathrm{eff}}\right)} \Omega_{m, 0}\left(1+z_{\mathrm{eff}}\right)^{3} \\
b(\hat{k}) & =b_{1}+b_{2} \hat{k}^{2}
\end{aligned}
\]

\section*{FS observational modelling}

Corrections for observational effects (window function) and systematics:
\[
\begin{aligned}
P_{g}^{\text {conv }}\left(k_{i}\right) & =\sum_{i j} W_{i j} P_{g}^{\mathrm{th}}\left(k_{j}\right)-\frac{\sum_{j} W_{0 j} P_{g}^{\mathrm{th}}\left(k_{j}\right)}{P_{w}(0)} P_{w}\left(k_{i}\right), \\
P_{g}^{\text {sys }}(k) & =P_{g}^{\text {conv }}(k)+S\left[P_{g}^{\text {meas }}(k)-P_{g}^{\text {nosys }}(k)\right] \\
\ln \mathcal{L}_{F S} & =-\frac{\Delta^{T} C^{-1} \Delta}{2}, \Delta \equiv P_{g}^{\text {meas }}-P_{g}^{\text {sys }}
\end{aligned}
\]

Follows Ross et al., MNRAS 428 (2013) 1116; Beutler et al., MNRAS 424 (2014) 564

\section*{Comparison to emulators}

Comparison to Coyote emulator


\section*{CC-only constraints}


\section*{CC systematics}
- Residual subdominant young population (i.e. tracer not unbiased)
- Star formation history uncertainties (not simple stellar populations)
- Stellar metallicity uncertainty (needed to calibrate relative ages)
- Stellar population synthesis model (many possible SPS models)

First three points already included in current uncertainty budgets, we took SPS uncertainty into account with redshift-dependent systematic budget following Moresco et al., ApJ 898 (2020) 82```


[^0]:    Credits: Planck public chains

