The trouble with spatial curvature: present, future, and model-independent determinations

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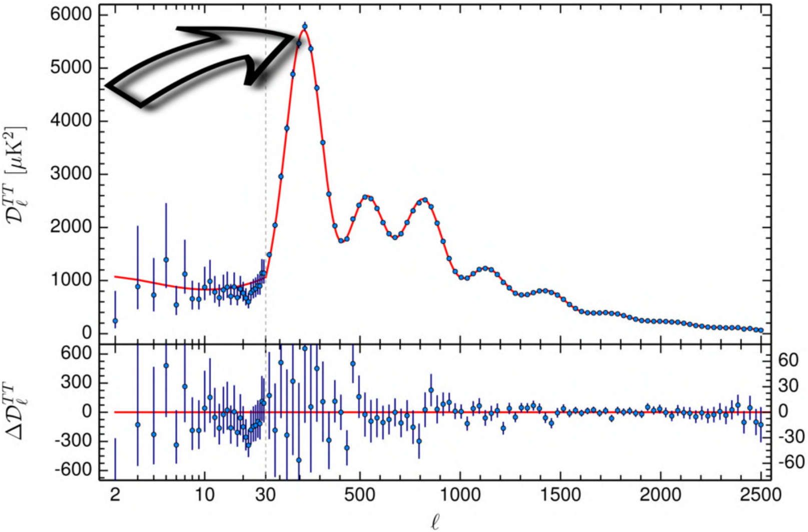
What is the shape of the Universe?

What is the sign of the spatial curvature parameter Ω_K?
Can we get model-independent constraints on Ω_K?

- It is true that *Planck* CMB temperature and polarization data appears to prefer a spatially closed Universe ($\Omega_K < 0$)
- However, to get a *reliable* constraint we must combine *Planck* with external data to break the *geometrical degeneracy* in a *reliable* way...
- ...from which we learn that the Universe is flat to the $\sim \mathcal{O}(10^{-2})$ level
- (and yes, one can infer Ω_K in a cosmology-free way!)

Planck 2018 temperature power spectrum



Credits: *Planck* collaboration, A& A 641 (2020) A6

The geometrical degeneracy

How far away is this person (hopefully more than 2m)? d
How tall is this person? h
Only data: angle subtended by this person $\theta \approx h/d$

You can't disentangle distance and height from this data alone: geometrical degeneracy!

Breaking the geometrical degeneracy

Answer: roughly 7m away and roughly 3m tall

How to break the geometrical degeneracy?

Need to pin down post-recombination expansion rate: Ω_m, H_0, $H(z)$,...

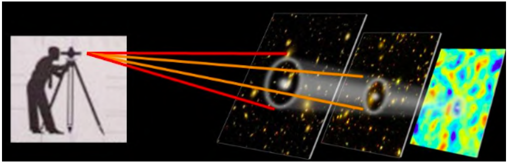
$$D_A(z) = \int_0^z \frac{dz'}{H(z')} \simeq \int_0^z \frac{dz'}{H_0\sqrt{\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + (1-\Omega_m-\Omega_K)}}$$

$$\downarrow$$

$$\theta_s = \frac{r_s(z_{\rm LS})}{D_A(z_{\rm LS})} = \frac{\int_{z_{\rm LS}}^{\infty} \frac{dz'}{H(z')}}{\int_0^{z_{\rm LS}} \frac{dz''}{H(z'')}} \longrightarrow \begin{cases} \theta_s(H_0\,, \Omega_m\,, \Omega_K) \approx 0.0104 \\ \Omega_m h^2 \approx 0.15 \\ D_i(z_i, H_0, \Omega_m\,, \Omega_K) = x_i \quad \text{or} \\ H_i(z_i, H_0, \Omega_m\,, \Omega_K) = y_i \end{cases}$$

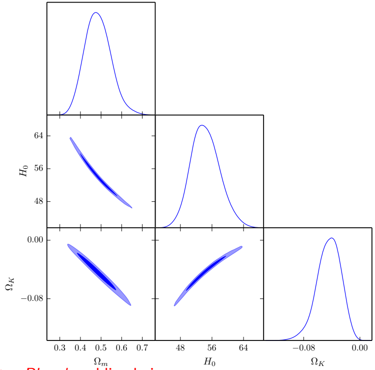
So a good way to break the geometrical degeneracy is to measure distances and expansion rates in the late-time Universe

CMB and BAO



Planck 2018 results (`Plik` likelihood)

$\Omega_K = -0.044^{+0.018}_{-0.015} \rightarrow$ apparent $\approx 3\sigma$ detection of $\Omega_K \neq 0$?



Credits: *Planck* public chains

Parameters \ Dataset	*Planck*
Ω_K	$-0.044^{+0.018}_{-0.015}$
H_0 [km/s/Mpc]	$54.36^{+3.25}_{-3.96}$
Ω_m	$0.485^{+0.058}_{-0.068}$

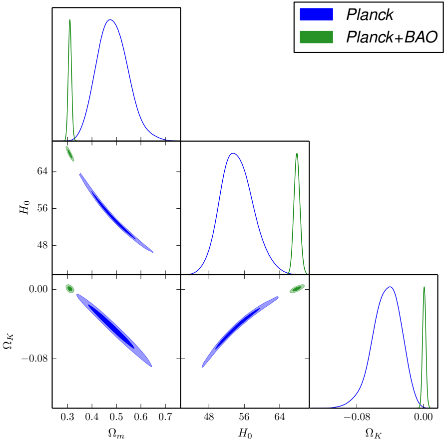
Implausible values of H_0 and Ω_m, excluded by any other independent local/late-time measurements

Related to $A_{\rm lens}$ problem, `CamSpec` analysis (with access to larger sky fraction) and *ACT DR4* results support possible fluke interpretation

Breaking the geometrical degeneracy

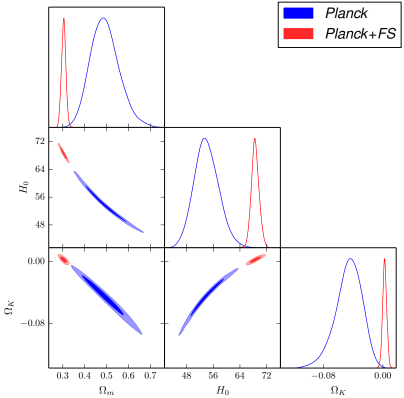
Examples: *Planck* TTTEEE+lowl+lowE

+BAO



Credits: *Planck* public chains

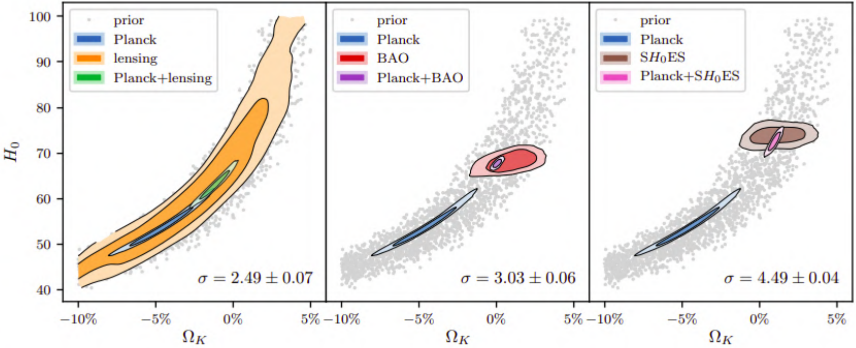
+full-shape galaxy power spectrum



SV, Di Valentino, Gariazzo, Melchiorri, Mena & Silk,

arXiv:2010.02230

Tensions with external datasets?



Credits: Handley, arXiv:1908.09139

Should we believe results coming from the combination of datasets in tension *within a given model*?
Can we break the geometrical degeneracy in a different way?

Breaking the geometrical degeneracy in an inconsistent way

An impasse?

- We want to break the geometrical degeneracy with external ("ext") datasets to stabilize *Planck* constraints on Ω_K
- *Planck*+ext always points towards $\Omega_K = 0$, but at the cost of significant tensions within ΛCDM+Ω_K
- Another problem: some of these external datasets (*e.g.* BAO and FS) carry some amount of model-dependence in the form of fiducial cosmological assumptions during data reduction process

How to exit this impasse?

Need a “golden dataset” which:

- helps to break the geometrical degeneracy once combined with *Planck* CMB temperature and polarization data
- is not in strong tension with *Planck* data when working within a non-flat Universe
- is as model-independent as possible

Cosmic chronometers

$$\frac{dt}{dz} = -\frac{1}{(1+z)H(z)}$$

Take two ensembles of passively evolving galaxies that formed at the same time and are separated by a small redshift interval Δz around $z_{\rm eff}$:

$$H(z_{\rm eff}) = -\frac{1}{1+z_{\rm eff}}\frac{\Delta z}{\Delta t}$$


THE ASTROPHYSICAL JOURNAL

Constraining Cosmological Parameters Based on Relative Galaxy Ages

Raul Jimenez[1] and Abraham Loeb[2]

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The Astrophysical Journal, Volume 573, Number 1

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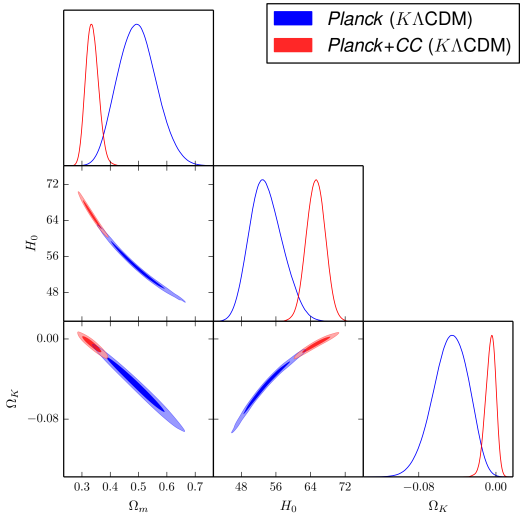
Jiménez & Loeb, ApJ 573 (2002) 37

Use massive, early-time, passively-evolving galaxies (evolving on a much longer timescale than their age differences)

Combining *Planck* with CC

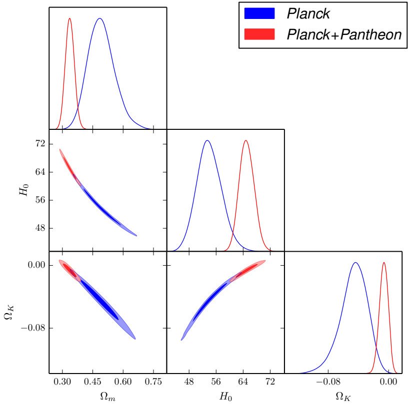
Planck+*CC*: $\Omega_K = -0.0054 \pm 0.0055 \rightarrow$ consistent with $\Omega_K = 0$ @$< 1\sigma$



SV, Loeb & Moresco, ApJ 908 (2021) 84

Combining *Planck* with *Pantheon*

Planck+*Pantheon* $\Omega_K = -0.0064 \pm 0.0058$



Caveat: 7-parameter ΛCDM+Ω_K, freeing w moves towards "phantom closed" Universe for both *Planck*+*CC* and *Planck*+*Pantheon* For *Planck*+*CC* see SV, Loeb & Moresco, ApJ 908 (2021) 84; for *Planck*+*Pantheon* see Di Valentino, Melchiorri & Silk, ApJL 908 (2021) L9

Model-independent constraints on spatial curvature

Assuming only FLRW metric:

$$H_0 d_L = \frac{c(1+z)}{\sqrt{|\Omega_K|}} \mathrm{sinn}\left(\sqrt{|\Omega_K|}\int_0^z \frac{dz'}{E(z')}\right)$$

$$\mathrm{sinn} = \begin{cases} \sin & \text{if } \Omega_K < 0 \\ \sinh & \text{if } \Omega_K > 0 \end{cases}$$

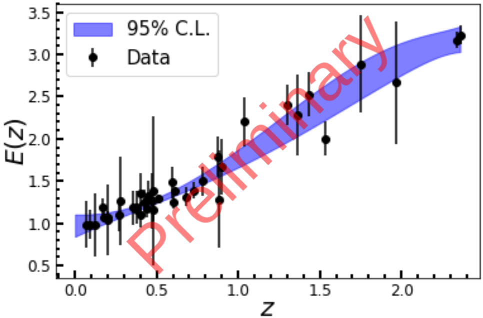
$H_0 d_L$: from *uncalibrated* (Hubble flow) SNeIa (*Pantheon*)
$E(z) = H(z)/H_0$: from cosmic chronometers

Dhawan, SV, Alsing, in preparation

Model-independent constraints on spatial curvature

Heuristically 2-step process:

- infer $E(z)$ from CC with GP regression
- use CC-inferred $E(z)$ and $H_0 d_L$ from SNeIa data to infer $\Omega_{\rm K}$ without assuming any cosmology



In reality: sample joint posterior of Ω_K, $H(z)$, $\mathcal{M}$, and GP hyperparameters, where the prior on $H(z)$ is a Gaussian process

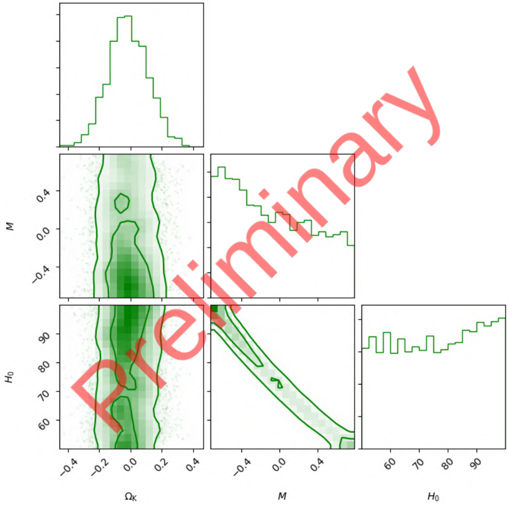
$\implies$ marginalize over all parameters (including GP hyperparameters)

$\implies$ infer Ω_K

Dhawan, SV, Alsing, in preparation

Model-independent constraints on spatial curvature

- With current data (*Pantheon* SNeIa, current CC): $\Omega_K = 0.02 \pm 0.25$
- With future data (SNeIa from NGRST, BAO from DESI, Euclid, NGRST, VRO): $\sigma_{\Omega_K} \sim 0.03$



Dhawan, SV, Alsing, in preparation

Conclusions

- Curvature parameter Ω_K is a key quantity in cosmology
- *Planck* temperature and polarization prefer $\Omega_K < 0$, but need to break geometrical degeneracy *without incurring in tensions*!
- Achieved combining *Planck* with cosmic chronometer or *Pantheon* Hubble flow SNeIa, combination consistent with spatial flatness
- Combining cosmic chronometers and Hubble flow SNeIa allows for a model-independent determination of Ω_K (currently to the 10^{-1} level, in the future to the 10^{-2} level)
- **Universe is spatially flat to the $\mathcal{O}(10^{-2})$ level**