The trouble with spatial curvature

Sunny Vagnozzi

Newton-Kavli Fellow @ KICC, University of Cambridge

✉ `sunny.vagnozzi@ast.cam.ac.uk` ⌂ `www.sunnyvagnozzi.com`
`sunnyvagnozzi` `@SunnyVagnozzi`

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Kavli Institute for Cosmology Cambridge KICC
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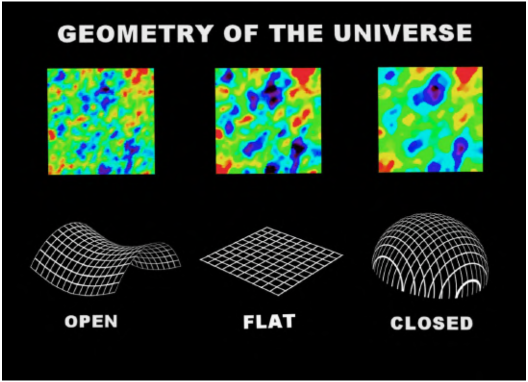
What is the shape of the Universe?

What is the shape of the Universe?
What is the sign of the spatial curvature parameter Ω_K?

- It is true that *Planck* CMB temperature and polarization data appears to prefer a spatially closed Universe ($\Omega_K < 0$)
- However, to learn more we must combine *Planck* data with external datasets to break the *geometrical degeneracy* in a *reliable* way...
- ...and doing so teaches us that the Universe is very likely spatially flat to the $|\Omega_K| \sim \mathcal{O}(10^{-2})$ level

What is the ~~shape~~ local geometry of the observable Universe?

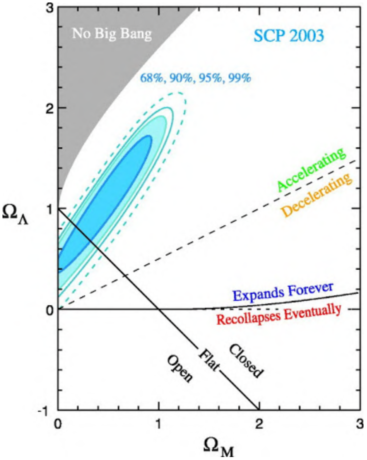
Credits: NASA/GSFC



$\Omega_K > 0$ $\Omega_K = 0$ $\Omega_K < 0$

The importance of spatial curvature

Late Universe: sign and value of Ω_K plays a key role in determining the future evolution of the Universe



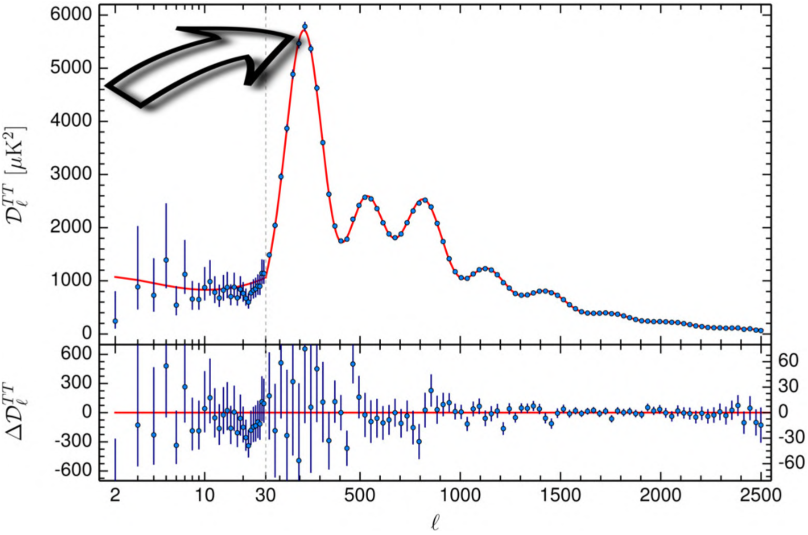
Early Universe: many inflation models predict (constructed to give) $\Omega_K \sim 0$

Measurement of $|\Omega_K| \gtrsim \mathcal{O}(10^{-4})$ would be a problem for many inflationary models

Generally easier to accommodate open rather than closed Universe from inflation

Credits: *Supernova Cosmology Project* collaboration

Planck 2018 temperature power spectrum



Credits: *Planck* collaboration, A& A 641 (2020) A6

The geometrical degeneracy

How far away is this person (hopefully more than 2m)? d
How tall is this person? h
Only data: angle subtended by this person $\theta \approx h/d$

You can't disentangle distance and height from this data alone: geometrical degeneracy!

Breaking the geometrical degeneracy

Answer: roughly 7m away and roughly 3m tall

The geometrical degeneracy

Key angular scale:

$$\theta_s = \frac{r_s(z_{\rm LS})}{D_A(z_{\rm LS})} = \frac{\int_{z_{\rm LS}}^{\infty} \frac{dz'}{H(z')}}{\int_0^{z_{\rm LS}} \frac{dz''}{H(z'')}}$$

Geometrical degeneracy notably affects Ω_K, H_0, and Ω_m (equivalently Ω_Λ)

Is the Universe:

- young (high H_0) with a large amount of vacuum energy and negative spatial curvature?
- spatially flat?
- old (low H_0) with little vacuum energy and positive spatial curvature?
- ...

How to break the geometrical degeneracy?

Need to pin down post-recombination expansion rate: Ω_m, H_0, $H(z)$,...

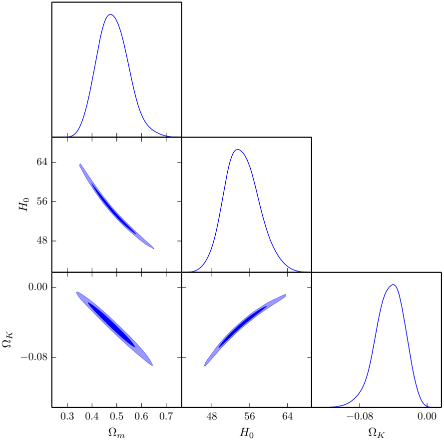
$$D_A(z) = \int_0^z \frac{dz'}{H(z')} \simeq \int_0^z \frac{dz'}{H_0\sqrt{\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + (1-\Omega_m-\Omega_K)}}$$

Examples:

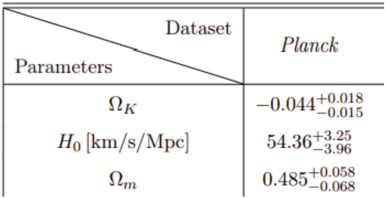
- BAO (D_V/r_s, D_A/r_s, Hr_s → help stabilizing Ω_m and H_0)
- CMB lensing (helps stabilizing Ω_m)
- Uncalibrated SNeIa (*Pantheon*, help stabilizing Ω_m)
- Local Cepheid- or TRGB-calibrated SNeIa measurements of H_0
- ++ (cluster counts, weak lensing, X-ray gas mass fraction,...)
- This talk: full-shape (FS) galaxy power spectrum → very closely related to BAO!
- This talk: cosmic chronometers (CC)

Planck 2018 results

Planck TTTEEE+lowE: $\Omega_K = -0.044^{+0.018}_{-0.015}$ → apparent detection of $\Omega_K \neq 0$ at the $\mathcal{O}(10^{-2} - 10^{-1})$ level?



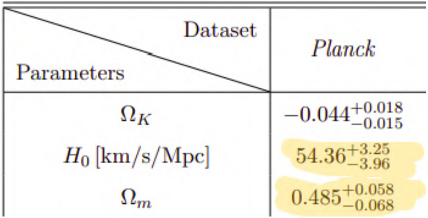
Credits: *Planck* public chains



Dataset Parameters	*Planck*
Ω_K	$-0.044^{+0.018}_{-0.015}$
H_0 [km/s/Mpc]	$54.36^{+3.25}_{-3.96}$
Ω_m	$0.485^{+0.058}_{-0.068}$

Planck 2018 results

Rather implausible (to say the least) values of H_0 and Ω_m within ΛCDM+Ω_K 7-parameter model ($K\Lambda$CDM)



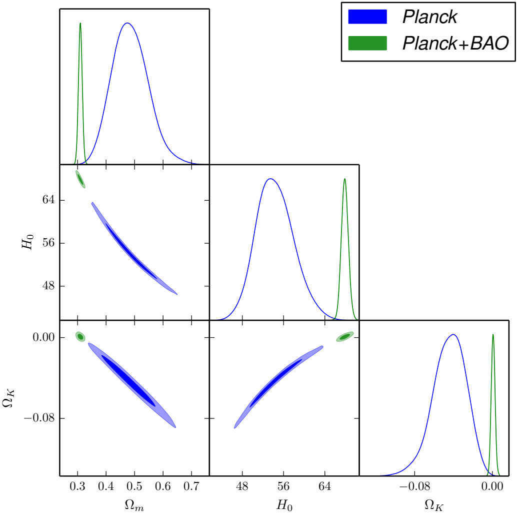
Dataset / Parameters	*Planck*
Ω_K	$-0.044^{+0.018}_{-0.015}$
H_0 [km/s/Mpc]	$54.36^{+3.25}_{-3.96}$
Ω_m	$0.485^{+0.058}_{-0.068}$

H_0 in strong tension with whatever local measurement you can think about (Cepheid- and TRGB-calibrated SNeIa, megamasers, H0LiCOW strong lensing,...)

Ω_m also in strong tension with late-time measurements (cosmic shear, cluster counts,...)

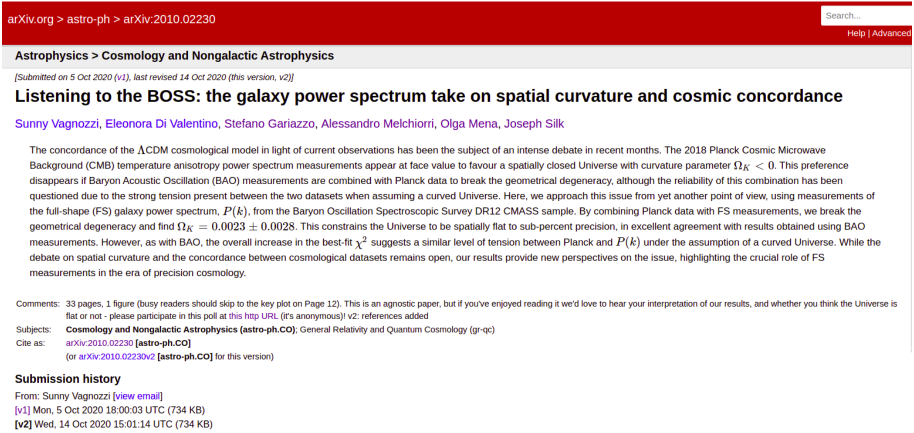
Breaking the geometrical degeneracy

Example: *Planck* TTTEEE+lowl+lowE+BAO



Credits: *Planck* public chains

Breaking the geometrical degeneracy with full-shape galaxy power spectrum data



Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 5 Oct 2020 (v1), last revised 14 Oct 2020 (this version, v2)]

Listening to the BOSS: the galaxy power spectrum take on spatial curvature and cosmic concordance

Sunny Vagnozzi, Eleonora Di Valentino, Stefano Gariazzo, Alessandro Melchiorri, Olga Mena, Joseph Silk

The concordance of the ΛCDM cosmological model in light of current observations has been the subject of an intense debate in recent months. The 2018 Planck Cosmic Microwave Background (CMB) temperature anisotropy power spectrum measurements appear at face value to favour a spatially closed Universe with curvature parameter $\Omega_K < 0$. This preference disappears if Baryon Acoustic Oscillation (BAO) measurements are combined with Planck data to break the geometrical degeneracy, although the reliability of this combination has been questioned due to the strong tension present between the two datasets when assuming a curved Universe. Here, we approach this issue from yet another point of view, using measurements of the full-shape (FS) galaxy power spectrum, $P(k)$, from the Baryon Oscillation Spectroscopic Survey DR12 CMASS sample. By combining Planck data with FS measurements, we break the geometrical degeneracy and find $\Omega_K = 0.0023 \pm 0.0028$. This constrains the Universe to be spatially flat to sub-percent precision, in excellent agreement with results obtained using BAO measurements. However, as with BAO, the overall increase in the best-fit χ^2 suggests a similar level of tension between Planck and $P(k)$ under the assumption of a curved Universe. While the debate on spatial curvature and the concordance between cosmological datasets remains open, our results provide new perspectives on the issue, highlighting the crucial role of FS measurements in the era of precision cosmology.

Comments: 33 pages, 1 figure (busy readers should skip to the key plot on Page 12). This is an agnostic paper, but if you've enjoyed reading it we'd love to hear your interpretation of our results, and whether you think the Universe is flat or not - please participate in this poll at this http URL (it's anonymous)! v2: references added

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc)

Cite as: arXiv:2010.02230 **[astro-ph.CO]**
(or arXiv:2010.02230v2 **[astro-ph.CO]** for this version)

Submission history

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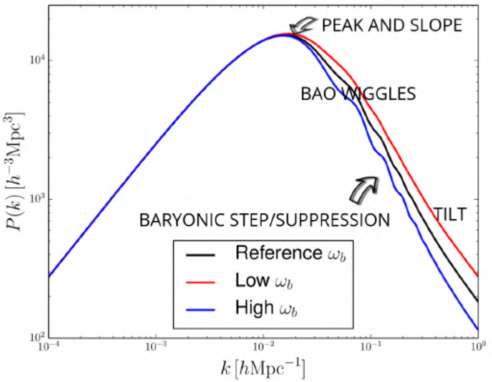
[v1] Mon, 5 Oct 2020 18:00:03 UTC (734 KB)
[v2] Wed, 14 Oct 2020 15:01:14 UTC (734 KB)

SV *et al.*, arXiv:2010.02230

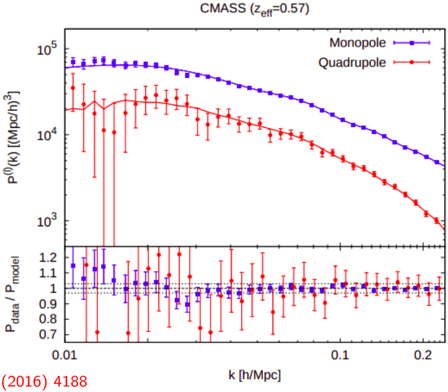
How can FS break the geometrical degeneracy?



- Position of BAO wiggles in k space → D_V → H_0
- k_{eq} turnaround in $P(k)$ → shape parameter $\Gamma \equiv \Omega_m h$
- Baryonic step/suppression → $\Omega_b h^2$ (hard to measure)
- The CMB already gives us $\Omega_m h^2$ → disentangle Ω_m and H_0

FS data

Monopole of pre-reconstructed BOSS DR12 CMASS power spectrum measured by Gil-Marín *et al.* [1] (conservative $k_{\text{max}} = 0.135\,h\,\mathrm{Mpc}^{-1}$ cutoff)



Gil-Marín *et al.*, MNRAS 460 (2016) 4188

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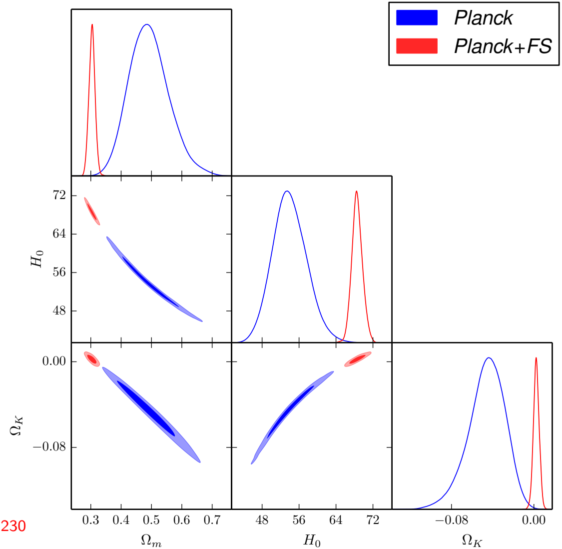
[1] Note: 1) not the same $P(k)$ quoted in "consensus" BOSS results (but gives consistent results); 2) not the same $P(k)$ used by recent EFTofLSS analyses

Combining *Planck* and FS data

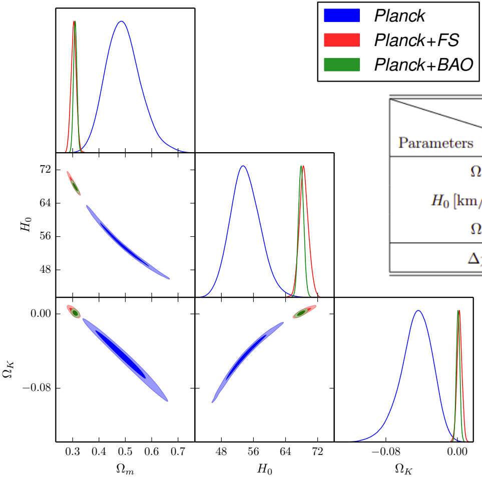
Planck+*FS*: $\Omega_K = 0.0023 \pm 0.0028 \rightarrow$ consistent with $\Omega_K = 0$ @$< 1\sigma$



SV *et al.*, arXiv:2010.02230

Compare FS and BAO

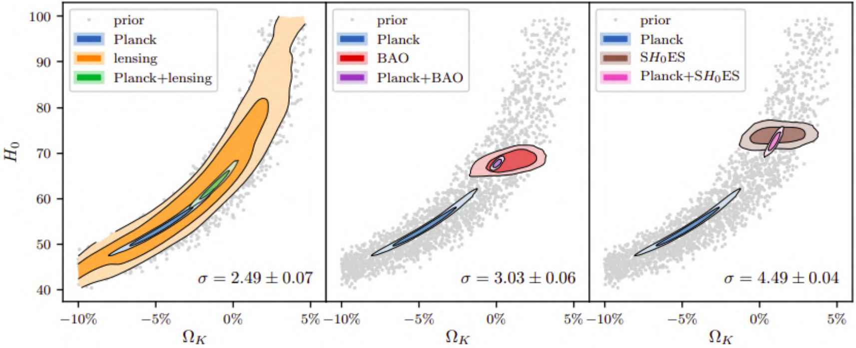
1. Consistent results across the two → good sanity check!
2. Sensible values for H_0 and Ω_m (also a good sanity check)
3. Much smaller $\Delta\chi^2$ (additional Ω_k parameter not preferred)



Parameters \ Dataset	*Planck*	*Planck+BAO*	*Planck+FS*
Ω_K	$-0.044^{+0.018}_{-0.015}$	0.0008 ± 0.0019	0.0023 ± 0.0028
H_0 [km/s/Mpc]	$54.36^{+3.25}_{-3.96}$	67.88 ± 0.66	$68.59^{+1.08}_{-1.20}$
Ω_m	$0.485^{+0.058}_{-0.068}$	0.310 ± 0.007	0.304 ± 0.010
$\Delta\chi^2$	-10.9	-0.6	-1.0

SV *et al.*, arXiv:2010.02230

Tensions with external datasets?



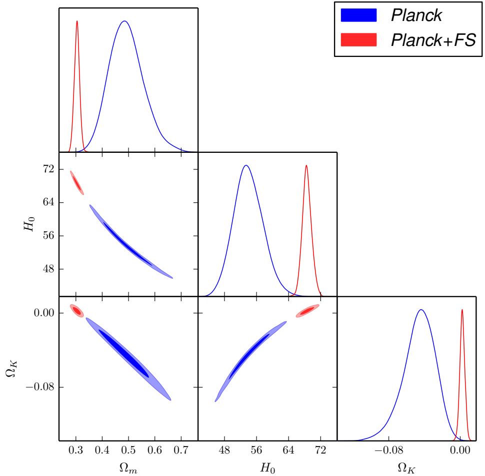
Credits: Handley, arXiv:1908.09139

Should we believe results coming from the combination of datasets in tension *within a given model*?

Can we break the geometrical degeneracy in a different way?

Tensions between *Planck* and FS?

We all see a 3σish tension by eye...



SV *et al.*, arXiv:2010.02230

An impasse?

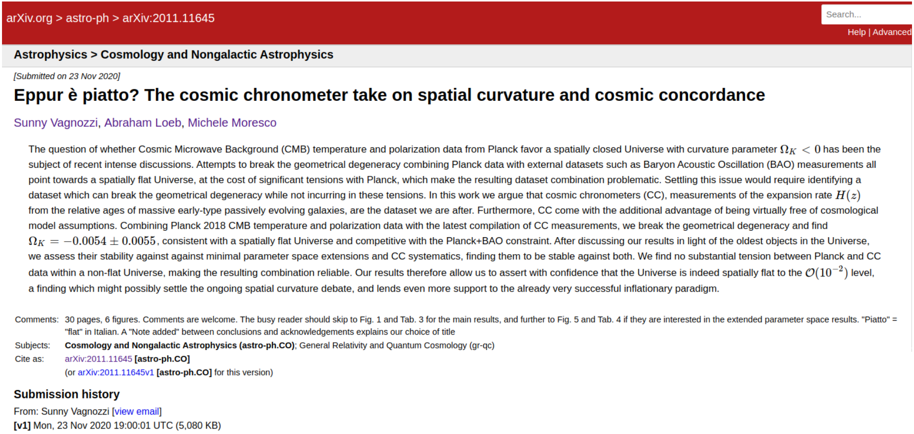
- We want to break the geometrical degeneracy with external datasets ("ext") to stabilize *Planck* constraints on Ω_K...
- ...but always run into tensions when doing so within $K\Lambda$CDM...
- ...including when using FS to break the geometrical degeneracy!
- *Planck*+ext always points towards $\Omega_K = 0$, including "ext"=FS
- Another problem: most of these external datasets (*e.g.* BAO and FS) carry some amount of model-dependence in the form of fiducial cosmological assumptions during data reduction process

How to exit this impasse?

Need a “golden dataset” which:

- helps to break the geometrical degeneracy once combined with *Planck* CMB temperature and polarization data
- is not in strong tension with *Planck* data when working within a non-flat Universe
- is as model-independent as possible

Cosmic chronometers to the rescue



Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 23 Nov 2020]

Eppur è piatto? The cosmic chronometer take on spatial curvature and cosmic concordance

Sunny Vagnozzi, Abraham Loeb, Michele Moresco

The question of whether Cosmic Microwave Background (CMB) temperature and polarization data from Planck favor a spatially closed Universe with curvature parameter $\Omega_K < 0$ has been the subject of recent intense discussions. Attempts to break the geometrical degeneracy combining Planck data with external datasets such as Baryon Acoustic Oscillation (BAO) measurements all point towards a spatially flat Universe, at the cost of significant tensions with Planck, which make the resulting dataset combination problematic. Settling this issue would require identifying a dataset which can break the geometrical degeneracy while not incurring in these tensions. In this work we argue that cosmic chronometers (CC), measurements of the expansion rate $H(z)$ from the relative ages of massive early-type passively evolving galaxies, are the dataset we are after. Furthermore, CC come with the additional advantage of being virtually free of cosmological model assumptions. Combining Planck 2018 CMB temperature and polarization data with the latest compilation of CC measurements, we break the geometrical degeneracy and find $\Omega_K = -0.0054 \pm 0.0055$, consistent with a spatially flat Universe and competitive with the Planck+BAO constraint. After discussing our results in light of the oldest objects in the Universe, we assess their stability against against minimal parameter space extensions and CC systematics, finding them to be stable against both. We find no substantial tension between Planck and CC data within a non-flat Universe, making the resulting combination reliable. Our results therefore allow us to assert with confidence that the Universe is indeed spatially flat to the $\mathcal{O}(10^{-2})$ level, a finding which might possibly settle the ongoing spatial curvature debate, and lends even more support to the already very successful inflationary paradigm.

Comments: 30 pages, 6 figures. Comments are welcome. The busy reader should skip to Fig. 1 and Tab. 3 for the main results, and further to Fig. 5 and Tab. 4 if they are interested in the extended parameter space results. "Piatto" = "flat" in Italian. A "Note added" between conclusions and acknowledgements explains our choice of title

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc)

Cite as: arXiv:2011.11645 **[astro-ph.CO]**
(or arXiv:2011.11645v1 **[astro-ph.CO]** for this version)

Submission history

From: Sunny Vagnozzi [view email]
[v1] Mon, 23 Nov 2020 19:00:01 UTC (5,080 KB)

SV *et al.*, arXiv:2011.11645

Cosmic chronometers

Age-redshift relation:

$$\frac{dt}{dz} = -\frac{1}{(1+z)H(z)}$$

Take two ensembles of passively evolving galaxies that formed at the same time and are separated by a small redshift interval Δz around z_{eff}:

$$H(z_{\mathrm{eff}}) = -\frac{1}{1+z_{\mathrm{eff}}}\frac{\Delta z}{\Delta t}$$

THE ASTROPHYSICAL JOURNAL

Constraining Cosmological Parameters Based on Relative Galaxy Ages

Raul Jimenez[1] and Abraham Loeb[2]

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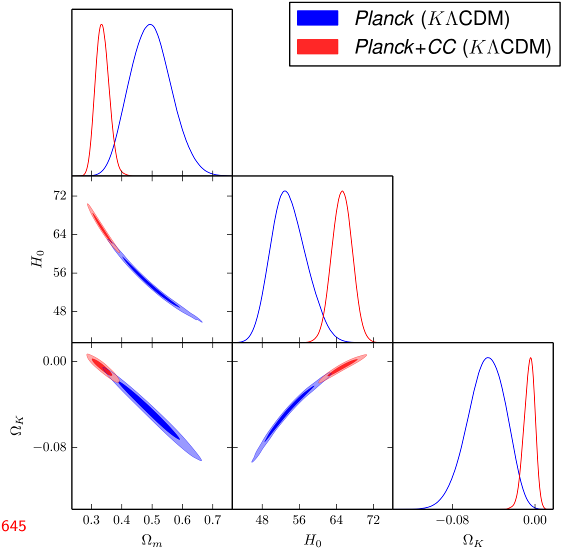
The Astrophysical Journal, Volume 573, Number 1

1393 Total downloads

Jiménez & Loeb, ApJ 573 (2002) 37

Combining *Planck* and CC data

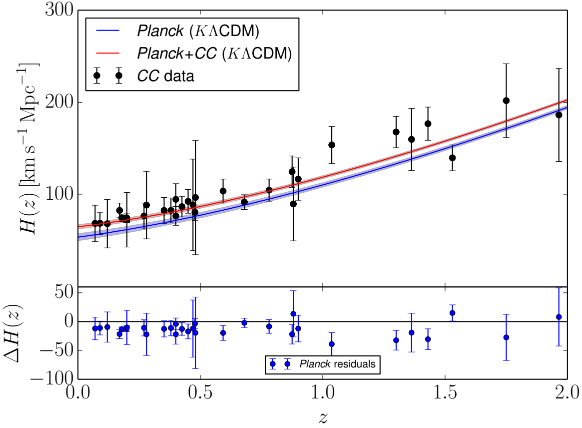
Planck+*CC*: $\Omega_K = -0.0054 \pm 0.0055 \rightarrow$ consistent with $\Omega_K = 0$ @$< 1\sigma$



SV *et al.*, arXiv:2011.11645

Tensions between *Planck* and CC?

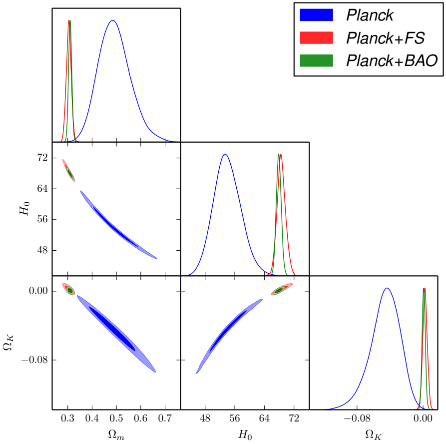
only a mild tension → we can trust the *Planck*+*CC* dataset combination even within a non-flat Universe



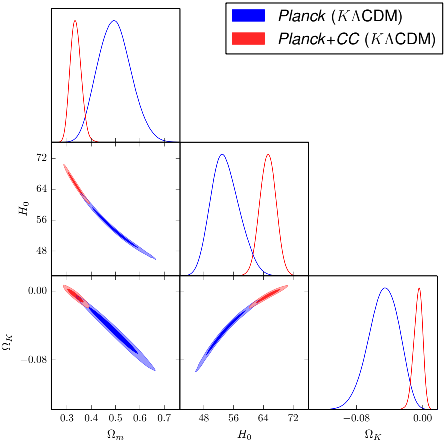
SV *et al.*, arXiv:2011.11645

Compare *Planck*+CC to *Planck*+BAO/FS

By eye much less tension, yet results still go towards $\Omega_K = 0$



SV *et al.*, arXiv:2010.02230



SV *et al.*, arXiv:2011.11645

Are cosmic chronometers our “golden dataset”?

Golden dataset characteristics:

- helps to break the geometrical degeneracy once combined with *Planck* CMB temperature and polarization data ✔
- is not in strong tension with *Planck* data when working within a non-flat Universe ✔
- is as model-independent as possible ✔

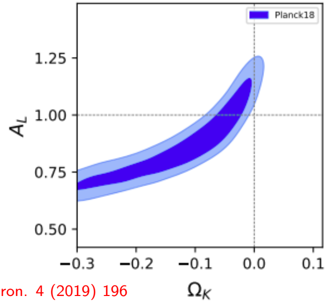
Conclusions

- Curvature parameter Ω_K is a key quantity in cosmology
- *Planck* CMB temperature and polarization data prefers $\Omega_K < 0$...
- but for a reliable result need to break geometrical degeneracy!
- Attempts to break the geometrical degeneracy incur in tensions...
- ...example: *Planck*+full-shape galaxy power spectrum data
  $\to \Omega_K = 0.0023 \pm 0.0028$ at the cost of a $\sim 3\sigma$ tension
- Cosmic chronometer data can break the geometrical degeneracy without incurring in strong tensions $\to \Omega_K = -0.0054 \pm 0.0055$
- **Universe is spatially flat to the $\mathcal{O}(10^{-2})$ level**

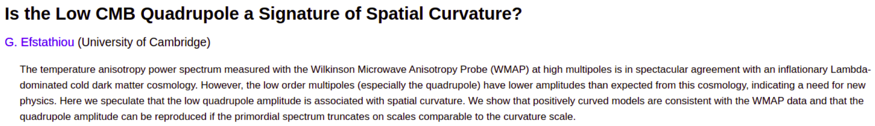
Backup slides

Where does this come from?

Partly (but not entirely) from the lensing/A_{lens} anomaly



Credits: Di Valentino *et al.*, Nat. Astron. 4 (2019) 196



Is the Low CMB Quadrupole a Signature of Spatial Curvature?

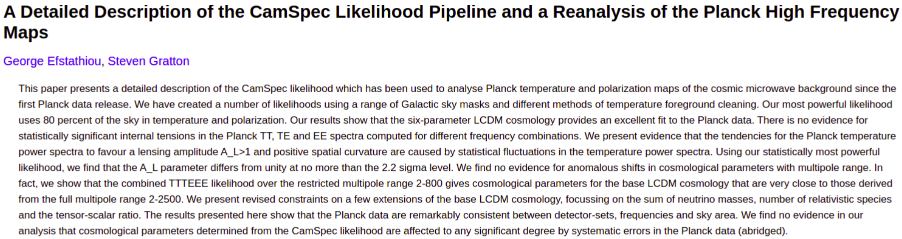
G. Efstathiou (University of Cambridge)

The temperature anisotropy power spectrum measured with the Wilkinson Microwave Anisotropy Probe (WMAP) at high multipoles is in spectacular agreement with an inflationary Lambda-dominated cold dark matter cosmology. However, the low order multipoles (especially the quadrupole) have lower amplitudes than expected from this cosmology, indicating a need for new physics. Here we speculate that the low quadrupole amplitude is associated with spatial curvature. We show that positively curved models are consistent with the WMAP data and that the quadrupole amplitude can be reproduced if the primordial spectrum truncates on scales comparable to the curvature scale.

Efstathiou, MNRAS 343 (2003) L95

Is this a fluke?

Significance of anomalies appears to decrease with more data (=access to higher sky fraction - using `12.5HMCl` `CamSpec` likelihood)...



A Detailed Description of the CamSpec Likelihood Pipeline and a Reanalysis of the Planck High Frequency Maps

George Efstathiou, Steven Gratton

This paper presents a detailed description of the CamSpec likelihood which has been used to analyse Planck temperature and polarization maps of the cosmic microwave background since the first Planck data release. We have created a number of likelihoods using a range of Galactic sky masks and different methods of temperature foreground cleaning. Our most powerful likelihood uses 80 percent of the sky in temperature and polarization. Our results show that the six-parameter LCDM cosmology provides an excellent fit to the Planck data. There is no evidence for statistically significant internal tensions in the Planck TT, TE and EE spectra computed for different frequency combinations. We present evidence that the tendencies for the Planck temperature power spectra to favour a lensing amplitude A_L>1 and positive spatial curvature are caused by statistical fluctuations in the temperature power spectra. Using our statistically most powerful likelihood, we find that the A_L parameter differs from unity at no more than the 2.2 sigma level. We find no evidence for anomalous shifts in cosmological parameters with multipole range. In fact, we show that the combined TTTEEE likelihood over the restricted multipole range 2-800 gives cosmological parameters for the base LCDM cosmology that are very close to those derived from the full multipole range 2-2500. We present revised constraints on a few extensions of the base LCDM cosmology, focussing on the sum of neutrino masses, number of relativistic species and the tensor-scalar ratio. The results presented here show that the Planck data are remarkably consistent between detector-sets, frequencies and sky area. We find no evidence in our analysis that cosmological parameters determined from the CamSpec likelihood are affected to any significant degree by systematic errors in the Planck data (abridged).

Efstathiou & Gratton, arXiv:1910.00483

...as one would expect if this were a fluke!

ACT DR4 (+WMAP) results consistent with $A_L = 1$ and $\Omega_K = 0$, no sign of lensing anomaly, support fluke interpretation Aiola *et al.*, arXiv:2007.07288

FS theoretical modelling

Alcock-Paczynski effect, RSD, Fingers-of-God, galaxy bias, shot noise:

$$\begin{aligned} P_g^{\mathrm{th}}(k, z_{\mathrm{eff}}) &= \frac{D_{A,\mathrm{fid}}^2(z_{\mathrm{eff}})}{D_A^2(z_{\mathrm{eff}})} \frac{H(z_{\mathrm{eff}})}{H_{\mathrm{fid}}(z_{\mathrm{eff}})} \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right) \exp\left[-\left(\hat{k}\sigma_{\mathrm{FoG}}\right)^2\right] \\ &\times\ b^2(\hat{k}) P_{m,\mathrm{HF}}(\hat{k}, z_{\mathrm{eff}}) + P_s \end{aligned}$$

where:

$$\begin{aligned} \hat{k} &= k \left[\frac{D_A^2(z_{\mathrm{eff}})}{D_{A,\mathrm{fid}}^2(z_{\mathrm{eff}})} \frac{H_{\mathrm{fid}}(z_{\mathrm{eff}})}{H(z_{\mathrm{eff}})}\right]^{\frac{1}{3}} \\ \beta(\hat{k}, z_{\mathrm{eff}}) &= \frac{f(\hat{k}, z_{\mathrm{eff}})}{b_0} = \frac{1}{b_0} \frac{d \ln \sqrt{P_m(\hat{k}, z_{\mathrm{eff}})}}{da} \\ f(\hat{k}, z_{\mathrm{eff}}) &\approx \Omega_m(z_{\mathrm{eff}})^{0.545} = \frac{H_0^2}{H^2(z_{\mathrm{eff}})} \Omega_{m,0}(1 + z_{\mathrm{eff}})^3 \\ b(\hat{k}) &= b_1 + b_2 \hat{k}^2 \end{aligned}$$

See also modelling for Euclid $P(k)$ forecasts in Sprenger *et al.*, JCAP 1902 (2019) 047

FS observational modelling

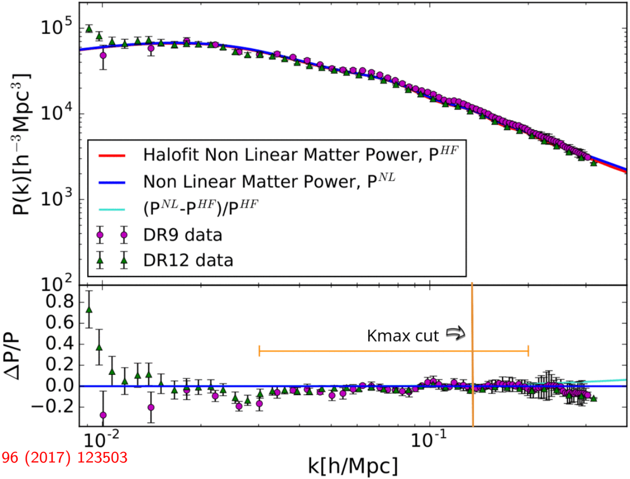
Corrections for observational effects (window function) and systematics:

$$\begin{aligned}
P_g^{\rm conv}(k_i) &= \sum_{ij} W_{ij} P_g^{\rm th}(k_j) - \frac{\sum_j W_{0j} P_g^{\rm th}(k_j)}{P_w(0)} P_w(k_i)\,, \\
P_g^{\rm sys}(k) &= P_g^{\rm conv}(k) + S\left[P_g^{\rm meas}(k) - P_g^{\rm nosys}(k)\right] \\
\ln \mathcal{L}_{FS} &= -\frac{\Delta^T C^{-1} \Delta}{2}\,, \quad \Delta \equiv P_g^{\rm meas} - P_g^{\rm sys}
\end{aligned}$$

Follows Ross *et al.*, MNRAS 428 (2013) 1116; Beutler *et al.*, MNRAS 424 (2014) 564

Comparison to emulators

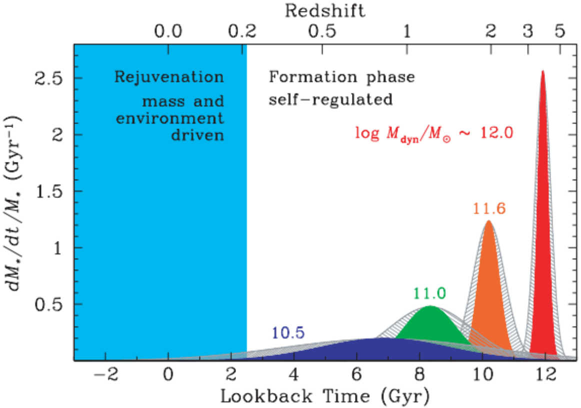
Comparison to Coyote emulator



SV *et al.*, PRD 96 (2017) 123503

Cosmic chronometers

Use massive, early-time, passively-evolving galaxies (evolving on a much longer timescale than their age differences)



Thomas, Maraston *et al.*, MNRAS 404 (2010) 1775

Advantages with respect to distance measurements

Luminosity/angular diameter distance:

$$D_L = (1+z)\int_0^z \frac{dz'}{H(z')} \qquad D_A = \frac{1}{1+z}\int_0^z \frac{dz'}{H(z')}$$

Distances suffer from integral sensitivity to expansion history and parameters such as the dark energy equation of state

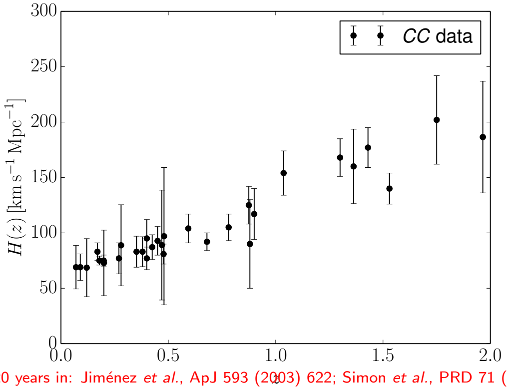
CMB acoustic scale:

$$\theta_s = \frac{r_s(z_{\rm LS})}{D_A(z_{\rm LS})} = \frac{\int_{z_{\rm LS}}^{\infty} \frac{dz'}{H(z')}}{\int_0^{z_{\rm LS}} \frac{dz''}{H(z'')}}$$

About half of the contribution to $D_A(z_{\rm LS})$ comes from $H(z)$ at $0 < z \lesssim 2$

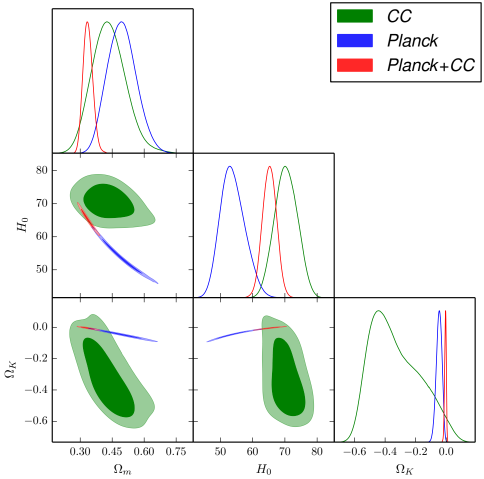
Cosmic chronometer measurements

Sweeping a lot of dust under the carpet, we'll assume these measurements are trustworthy See lots of works in the last 10 years, especially by Michele Moresco



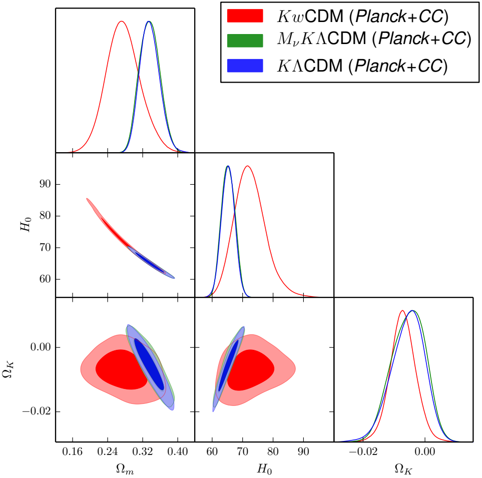
Compiled across the last 20 years in: Jiménez *et al.*, ApJ 593 (2003) 622; Simon *et al.*, PRD 71 (2005) 123001; Stern *et al.*, JCAP 1002 (2010) 008; Moresco *et al.*, JCAP 1207 (2012) 053; Zhang *et al.*, Res. Astron. Astrophy. 14 (2014) 1221; Moresco, MNRAS 450 (2015) L16; Moresco *et al.*, JCAP 1605 (2016) 014; Ratsimbazafy *et al.*, MNRAS 467 (2017) 3239

CC-only constraints



Are these results stable against an enlarged parameter space?

Yes (at least when varying w or M_ν)!



SV *et al.*, arXiv:2011.11645

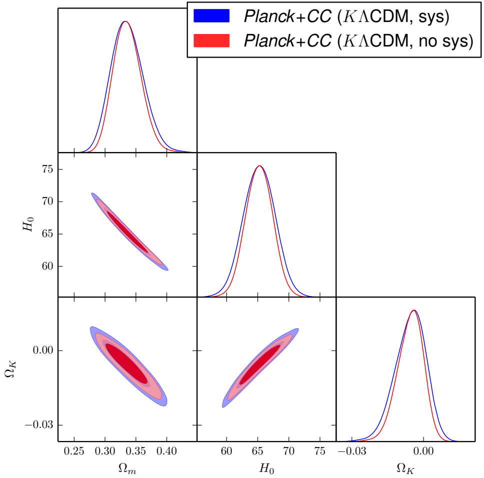
CC systematics

- Residual subdominant young population (*i.e.* tracer not unbiased)
- Star formation history uncertainties (not simple stellar populations)
- Stellar metallicity uncertainty (needed to calibrate relative ages)
- Stellar population synthesis model (many possible SPS models)

First three points already included in current uncertainty budgets, we took SPS uncertainty into account with redshift-dependent systematic budget following Moresco *et al.*, ApJ 898 (2020) 82

How much are these results affected by CC systematics?

Very little ($\lesssim 10\%$)! See Moresco *et al.*, ApJ 898 (2020) 82 for systematics study



SV *et al.*, arXiv:2011.11645