*Formulae compilation for Cosmology and Astroparticle Physics FK7050*
*Edvard Mörtsell*

## Constants

| Quantity | Symbol | Value (SI-units) |
|---|---|---|
| Speed of light | $c$ | $2.998 \cdot 10^8$ m/s |
| Newton's constant | $G_\mathrm{N}$ | $6.672 \cdot 10^{-11}$ N m$^2$/kg$^2$ |
| Planck's constant | $\hbar = h/2\pi$ | $1.055 \cdot 10^{-34}$ Js |
| Stefan-Boltzmann constant | $\sigma$ | $5.670 \cdot 10^{-8}$ W/(m$^2$ K$^4$) |
| Boltzmann constant | $k_\mathrm{B}$ | $1.381 \cdot 10^{-23}$ J/K |
| Fine structure constant | $\alpha$ | $7.297 \cdot 10^{-3}$ |
| Thomson cross section | $\alpha_\mathrm{T}$ | $6.652 \cdot 10^{-29}$ m$^2$ |

## Conversions

| |
|---|
| 1 erg = $10^{-7}$ J |
| 1 AU = $1.496 \cdot 10^{11}$ m |
| 1 pc = $3.086 \cdot 10^{16}$ m |
| 1 year = $3.156 \cdot 10^7$ s |
| $x$ K = $(x - 273.15)$ °C |
| 1 eV = $1.602 \cdot 10^{-19}$ J |

## Natural units

Setting $c = \hbar = k_\mathrm{B} = 1$, all dimensionful quantities can be written in units of, e.g. GeV.

| | |
|---|---|
| 1 GeV = | $1.1605 \cdot 10^{13}$ K |
| = | $1.7827 \cdot 10^{-27}$ kg |
| = | $(1.9733 \cdot 10^{-16}\,\mathrm{m})^{-1}$ |
| = | $(6.5821 \cdot 10^{-25}\,\mathrm{s})^{-1}$ |
| Stefan-Boltzmann constant | $\sigma = \pi^2/60$ |
| Newton's constant | $G_\mathrm{N} = m_\mathrm{Pl}^{-2} = 6.7076 \cdot 10^{-39}$ GeV$^{-2}$ |

## Masses

| Quantity | Symbol | Value |
|---|---|---|
| Electron mass | $m_{\rm e}c^2$ | $5.110 \cdot 10^5$ eV |
| Proton mass | $m_{\rm p}c^2$ | $9.383 \cdot 10^8$ eV |
| Neutron mass | $m_{\rm n}c^2$ | $9.396 \cdot 10^8$ eV |
| Sun's mass | $M_\odot$ | $1.989 \cdot 10^{30}$ kg |
| Earth's mass | $M_{\rm E}$ | $5.977 \cdot 10^{24}$ kg |

## Useful estimates

The following estimates are typically good for (close to) two significant numbers accuracy. The dimensionless Hubble constant is defined as $h \equiv H_0/(100$ km/s/Mpc).

| Quantity | Symbol | Value |
|---|---|---|
| Speed of light | $c$ | $3 \cdot 10^5$ km/s |
| Fine structure constant | $\alpha$ | 1/137 |
| Proton/neutron mass | $m_{\rm p,n}c^2$ | 1 GeV |
| Sun's mass | $M_\odot$ | $2 \cdot 10^{30}$ kg |
| Sun's Schwarzschild radius | $2GM_\odot/c^2$ | 3 km |
| Sun's luminosity | $L_\odot$ | $4 \cdot 10^{26}$ W |
| Earth's mass | $M_{\rm E}$ | $6 \cdot 10^{24}$ kg |
| Year | yr | $\pi \cdot 10^7$ s |
| Megaparsec | Mpc | $3 \cdot 10^{19}$ km |
| Hubble radius | $r_{\rm H} \equiv c/H_0$ | 3/h Gpc |
| Hubble time | $t_{\rm H} \equiv 1/H_0$ | $10^{10}$/h years |
| Critical density | $\rho_{\rm Crit} \equiv 3H_0^2/(8\pi G)$ | $2 \cdot 10^{-26}h^2$ kg/m$^3$ |
| | | $3 \cdot 10^{11}h^2 M_\odot/{\rm Mpc}^3$ |

## Equations

Christoffel symbol: $\Gamma^\sigma_{\mu\nu} = \frac{g^{\rho\sigma}}{2}\left(\frac{\delta g_{\nu\rho}}{\delta x^\mu} + \frac{\delta g_{\mu\rho}}{\delta x^\nu} + \frac{\delta g_{\mu\nu}}{\delta x^\rho}\right)$

Riemann tensor: $R_{\alpha\mu\gamma\nu} = g_{\alpha\beta}\left(\frac{\delta \Gamma^\beta_{\mu\nu}}{\delta x^\gamma} - \frac{\delta \Gamma^\beta_{\mu\gamma}}{\delta x^\mu} + \Gamma^\beta_{\rho\gamma}\Gamma^\rho_{\mu\nu} - \Gamma^\beta_{\rho\nu}\Gamma^\rho_{\mu\gamma}\right)$

Ricci tensor: $R_{\mu\nu} = g^{\alpha\gamma}R_{\alpha\mu\gamma\nu}$

Ricci scalar: $R = g^{\mu\nu}R_{\mu\nu}$

Einstein tensor: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$

Field equations of general relativity:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Perfect fluid stress-energy tensor:

$$T^{\mu\nu} = (p + \rho c^2) u^\mu u^\nu - p g^{\mu\nu}$$

Friedmann-Robertson-Walker-Lemaître line element:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right)$$

$$k = \begin{cases} 1 & \text{(closed)} \\ -1 & \text{(open)} \\ 0 & \text{(flat)} \end{cases}$$

Friedmann, acceleration and conservation equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} = H_0^2 \left[\Omega_{\mathrm{M}}(1+z)^3 + \Omega_{\mathrm{k}}(1+z)^2 + \Omega_\Lambda\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = H_0^2 \left[-\frac{\Omega_{\mathrm{M}}}{2}(1+z)^3 + \Omega_\Lambda\right]$$

$$\frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}a^3$$

Redshift-time relation:

$$dt = -\frac{dz}{H(z)(1+z)}$$

Energy density at radiation dominated epochs:

$$\rho_{\mathrm{R}} = \frac{\pi^2}{30} g_{\mathrm{eff}} T^4$$

Proper, angular and luminosity distances:

$$d_p(t_0) = \int_0^z \frac{dz}{H(z)}$$

$$d_A = \frac{a_0}{1+z} S\left(\frac{d_p(t_0)}{a_0}\right)$$

$$d_L = (1+z)^2 d_A$$

$$S(x) = \begin{cases} \sin(x) & k = 1 \\ x & k = 0 \\ \sinh(x) & k = -1 \end{cases}$$

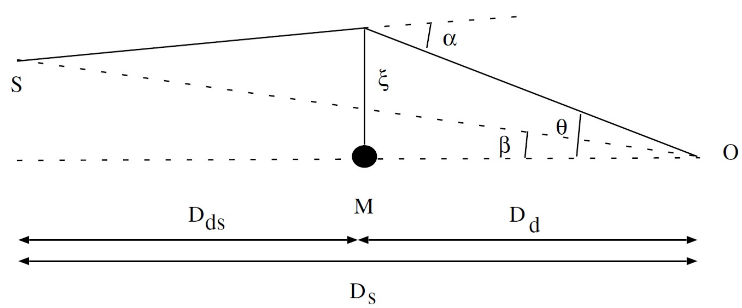
Magnitude-distance relation:

$$m = M + 5\log\left(\frac{d_L}{10\,\mathrm{pc}}\right)$$

Linear (one-component universe) structure formation:

$$\ddot{\delta} + 2H\dot{\delta} + \left(\frac{k^2 v_s^2}{a^2} - \frac{3H^2}{2}\right)\delta = 0$$

Gravitational lens equation:




$$\beta = \theta - \alpha\frac{D_{ds}}{D_s}$$

$$\alpha = \frac{4GM}{\xi} = \frac{2r_s}{\xi} = \frac{4GM}{D_d\theta}$$