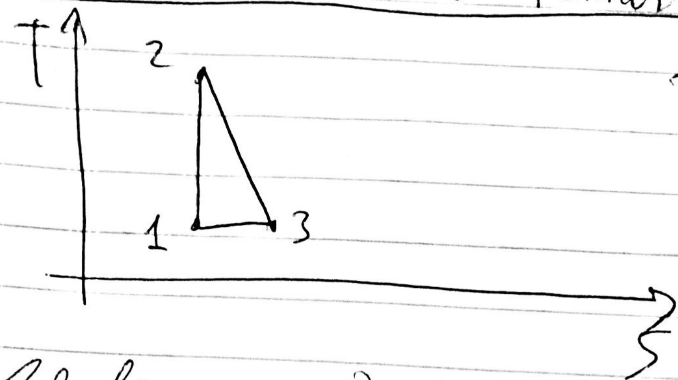
SOLUZIONI ESAME TERMODINAMICA FEBBRAIO 2024

$\boxed{1}$



$$\frac{T_2}{T_1} = \frac{T_2}{T_3} = 3$$

$$\frac{S_3}{S_1} = \frac{S_3}{S_2} = 2$$

Calcoliamo $Q, W, \Delta U$ ramo per ramo

$1 \to 2$

$Q_{12} = \int_1^2 T\,dS = 0$ $\qquad \Delta U = Q - W$

$\Delta U_{12} = n c_v \Delta T = \frac{5}{2} nR (3T_1 - T_1) = +5nRT_1$

$W_{12} = Q_{12} - \Delta U_{12} = -5nRT_1$

$2 \to 3$

$Q_{23} = \text{area trapezio} = \frac{(T_2+T_1)(S_3-S_1)}{2} = \frac{4T_1S_1}{2} = 2T_1S_1$

$Q_{23} > 0 \to Q_{in}$

$\Delta U_{23} = n c_v \Delta T = \frac{5}{2} nR(T_1 - 3T_1) = -5nRT_1$

$W_{23} = Q_{23} - \Delta U_{23} = 2T_1S_1 + 5nRT_1$

$3 \to 1$

$Q_{31} = \int_3^1 T\,dS = T_1(S_1 - S_3) = T_1(S_1 - 2S_1) = -T_1S_1$

$Q_{31} < 0 \Rightarrow Q_{out}$

$\Delta U_{31} = n c_v \Delta T = 0$ $\qquad W_{31} = Q_{33} - \Delta U_{31} = -T_1S_1$

$Q_{tot} = T_1S_1$ $\qquad W_{tot} = T_1S_1 \to \Delta U_{tot} = 0$ ✓

$Q_{in} = 2T_1S_1$ $\qquad Q_{out} = -T_1S_1$

a) $\eta = \frac{W_{tot}}{Q_{in}} = \frac{T_1 S_1}{2T_1 S_1} = \frac{1}{2}$ o $\eta = 1 - \frac{|Q_{out}|}{Q_{in}} = 1 - \frac{T_1 S_1}{2T_1 S_1} = 1 - \frac{1}{2} = \frac{1}{2}$

b) $\eta_{carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_1}{3T_1} = 1 - \frac{1}{3} = \frac{2}{3}$ $\quad \frac{1}{2} < \frac{2}{3}$

c) Siccome tutti i rami sono reversibili, $\Delta S_{amb} = -\Delta S_{gas}$

$1 \to 2 \quad \Delta S_{gas} = 0 \to \Delta S_{amb} = 0$

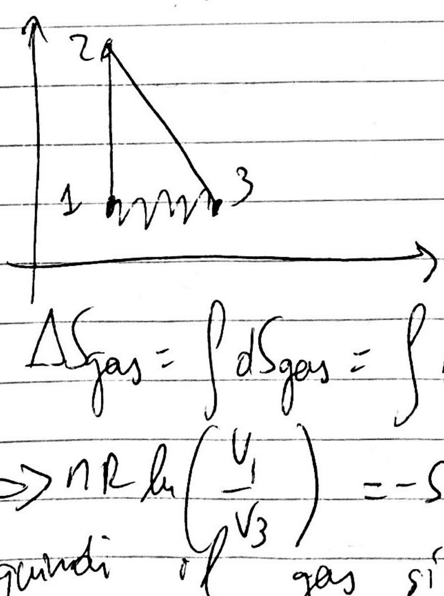
$2 \to 3 \quad \Delta S_{gas} = +S_1 \to \Delta S_{amb} = \cancel{\Delta S_1} \; -S_1$

$3 \to 1 \quad \Delta S_{gas} = -S_1 \to \Delta S_{amb} = \cancel{\Delta S_1} \; +S_1$

d) È banalmente 0 in quanto l'ambiente circostante ausiliario per definizione non partecipa alle trasformazioni

e) non cambierebbe niente in quanto W_{tot}, Q_{in}, Q_{out} non cambiano
W sui singoli rami dipende da C_v $\left(= \frac{3}{2}R \text{ o } \frac{5}{2}R\right)$ ma W_{tot} no!

f)



$dS_{gas} = \cancel{n c_v \frac{dT}{T}}^{\,T = const} + nR\frac{dV}{V} = nR\frac{dV}{V}$

$\Rightarrow \Delta S_{gas} = S_1 - S_3 = -S_1$

$\Delta S_{gas} = \int dS_{gas} = \int nR\frac{dV}{V} = nR \ln\left(\frac{V_1}{V_3}\right)$

$\Rightarrow nR\ln\left(\frac{V_1}{V_3}\right) = -S_1 < 0 \to \ln\left(\frac{V_1}{V_3}\right) < 0 \to V_1 < V_3$

quindi il gas si contrae

$nR\ln\left(\frac{V_1}{V_3}\right) = -S_1 \to \boxed{\frac{V_1}{V_3} = e^{-\frac{S_1}{nR}}}$

$T = const \Rightarrow PV = const \Rightarrow \frac{P_1V_1}{P_3V_3} = \cancel{const}\, 1 \Rightarrow \boxed{\frac{P_1}{P_3} = \frac{V_3}{V_1} = e^{\frac{S_1}{nR}}}$

$\boxed{2}$ a) $T(V) = \alpha V$ $\quad V_i \to V_f = \beta V_i$

$T_i = \alpha V_i \to T_f = \alpha V_f = \alpha \beta V_i = \beta T_i$

$PV = nRT \to P = \frac{nRT}{V} = \frac{nR\alpha V}{V} = nR\alpha = \text{const}$

$W = \int dV\, P(V) = nR\alpha \int dV = nR\alpha (V_f - V_i) =$

$= nR\alpha (\beta V_i - V_i) = nR\alpha (\beta - 1) V_i = nR\cancel{\alpha}(\beta-1)\frac{T_i}{\cancel{\alpha}} = nR(\beta-1)T_i$

$\Delta U = nC_V \Delta T = \frac{5}{2} nR (T_f - T_i) = \frac{5}{2} nR (\beta - 1) T_i$

$Q = \Delta U + W = nR(\beta-1)T_i + \frac{5}{2} nR(\beta-1)T_i = \frac{7}{2} nR (\beta-1) T_i$

b) $T(P) = \alpha P$ $\quad P_i \to P_f = \beta P_i$

$T_i \to T_f = \beta T_i$ come sopra

$PV = nRT \to V = \frac{nRT}{P} = \frac{nR\alpha P}{P} = nR\alpha = \text{const}$

$W = \int dV\, P(V) = 0$

$\Delta U = nC_V \Delta T = \frac{5}{2} nR (T_f - T_i) = \frac{5}{2} nR (\beta - 1) T_i$

$Q = \Delta U + W = \frac{5}{2} nR (\beta - 1) T_i$

c) $T(S) = \alpha S$ $\quad S_i \to S_f = \beta S_i$

$T_i \to T_f = \beta T_i$ come sopra

$Q = \int dS\, T(S) = \int_{S_i}^{S_f} dS\, \alpha S = \alpha \frac{S^2}{2}\Big|_{S_i}^{S_f} = \frac{\alpha}{2}(S_f^2 - S_i^2) =$

$= \frac{\alpha}{2}(\beta^2 S_i^2 - S_i^2) = \frac{\alpha}{2}(\beta^2 - 1) S_i^2 = \frac{\alpha}{2}(\beta^2-1)\frac{T_i^2}{\alpha^2} = \frac{(\beta^2-1)}{2\alpha} T_i^2$

$$\Delta U = nc_v \Delta T = \frac{5}{2} nR (T_f - T_i) = \frac{5}{2} nR(\beta - 1) T_i$$

$$W = Q - \Delta U = \frac{(\beta^2 - 1)}{2\alpha} T_i^2 - \frac{5}{2} nR (\beta - 1) T_i$$

Per quanto riguarda ΔS_{amb}, questa è $\Delta S_{amb} = -\Delta S_{gas}$ per tutti e tre i casi essendo le trasformazioni reversibili

a) $\Delta S_{gas} = \int dS_{gas} = \int nc_v \frac{dT}{T} + \int nR \frac{dV}{V} = nc_v \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$
$= nc_v \ln\beta + nR\ln\beta = \frac{7}{2} nR \ln\beta$

$$\Longrightarrow \Delta S_{amb} = -\frac{7}{2} nR\ln\beta$$

$V_i = V_f$

b) $\Delta S_{gas} = nc_v \ln\left(\frac{T_f}{T_i}\right) + \cancel{nR\ln\left(\frac{V_f}{V_i}\right)} = nc_v \ln\beta = \frac{5}{2} nR\ln\beta$

$$\Delta S_{amb} = -\frac{5}{2} nR \ln\beta$$

c) Banalmente $\Delta S_{gas} = S_f - S_i = (\beta - 1) S_i \Rightarrow \Delta S_{amb} = (1-\beta) S_i$

$\boxed{3}$ ~~$\mathcal{P}(=100\,W)$ $N(=100)$ $V = xyz$ $c_v = \frac{5}{2}R$~~

Pareti rigide $\longrightarrow$ trasformazione ~ isocora (**non** adiabatica perché c'è scambio di calore aria $\Longleftrightarrow$ uccellini)

$T_i = 20°C = 293.15\,K$

$T_f = 55°C = 328.15\,K$

$p_i = p_{atm} \sim 1.01325 \times 10^5\ Pa$

$\mathcal{P}\ (= 100\,W \quad o \quad 1000\,W)$

$N (= 100 \quad o \quad 30)$

$V = xyz = (20 \times 20 \times 3)\, m^3$

$$Q = N\mathcal{P}\Delta t = \frac{5}{2} nR\Delta T \Rightarrow \Delta t = \frac{5nR\Delta T}{2N\mathcal{P}}$$

ma $P_i V = nRT_i \longrightarrow n = \frac{P_i V}{RT_i}$

$$\Delta t = \frac{5nR\Delta T}{2N\mathcal{P}} = \frac{5P_iV\cancel{R}\Delta T}{2\cancel{R}T_iN\mathcal{P}} = \frac{5P_iV\Delta T}{2T_iN\mathcal{P}} =$$

$$= \frac{5P_i\,xyz\,(T_f - T_i)}{2T_iN\mathcal{P}}$$

Controllo unità

$$[\ \ ] = \frac{Pa\cdot m\cdot m\cdot m\cdot K}{K\cdot W} = \frac{\cancel{kg}\cdot \cancel{m}}{\cancel{m}\cdot s^2}\cdot\frac{\cancel{m}\cdot\cancel{m}\cdot\cancel{m}\cdot\cancel{K}}{\cancel{K}\cdot\cancel{kg}\cdot\cancel{m^2}}\cdot s^3$$

$= s$ ✓

Quindi $\Delta t = \frac{5P_i\,xyz\,(T_f - T_i)}{2T_iN\mathcal{P}}$ in s

a) Mettendo i numeri viene $\Delta t = 3629.4s \sim 61\ min$
Quindi muoiono alle 21:01

b) $\Delta t = 12097.5s \sim 202\ min = 3hr\ 22\ min$
Quindi muoiono alle 23:22

c) $\Delta t = 362.926s \sim 6\ min$
Quindi muoiono alle 20:06

d) Ovviamente $\Delta U_{tot} = 0$ perché $Q_{tot} = 0$ (pareti adiabatiche) e $W_{tot} = 0$ (pareti rigide), il calore scambiato solo internamente al sistema

e) L'ambiente esterno avrebbe rallentato il riscaldamento (assumendo che fuori facesse <55°!)