$\eta = 1 - \frac{T_L}{T_H}$ rendimento ideale del ciclo di Carnot, non dipende da tipo di gas (ideale) e dimensioni del sistema

Se $T_L \sim T_H$ $\quad \eta \ll 1$

Se $T_L \ll T_H$ $\quad \eta \sim 1$ $\quad$ ($\eta \to 1$ per $T_L \to 0$ e/o $T_H \to \infty$)

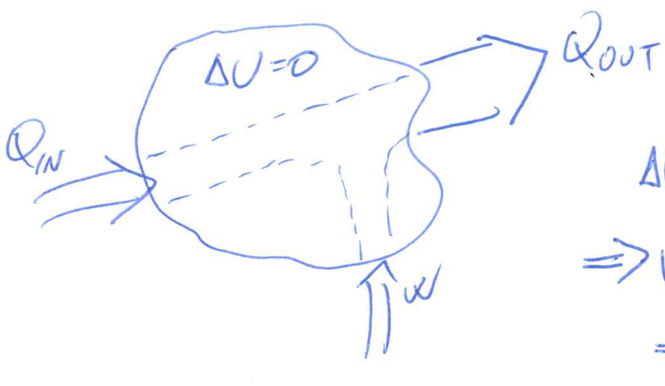
Supponiamo $T_L \sim 300 k$ (ambiente!)
$T_H \sim 600 k$ $\quad \to \eta = 0.5$

$T_L \sim 273.15 k$ (punto del ghiaccio)
$T_H \sim 373.15 k$ (punto del vapore) $\quad \to \eta \simeq 0.27$

Macchine frigorifere

In precedenza avevamo, per un ciclo, $\Delta U = 0$ con $Q = W$, ma $Q_{IN} \geq |Q_{OUT}|$ così che $W = Q_{IN} - |Q_{OUT}| > 0$ ovvero lavoro netto positivo: estraiamo energia meccanica dal sistema "pompando" calore

Possiamo lavorare "al contrario", immettere energia meccanica nel sistema (lavoro negativo), purché si estrae più di calore di quello che viene assorbito



$\Delta U = 0 = Q - W = Q_{IN} + Q_{OUT} - W = 0$

$\Rightarrow W = Q_{IN} + Q_{OUT} < 0$

$Q_{IN} < |Q_{OUT}|$

$\Rightarrow Q_{IN} < \underbrace{-Q_{OUT}}_{Q_{OUT} < 0} = |Q_{OUT}|$

Usiamo una macchina frigorifera per estrarre energia termica da un ~~serbatoio~~ "freddo" a spese di lavoro compiuto dal sistema.

→ ceduta a un serbatoio "caldo"!

Frigorifero efficiente ⇔ preleva molto Q_{IN} utilizzando poco W

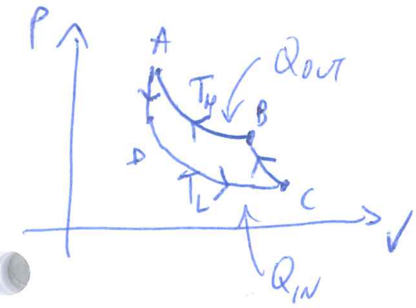
Coefficiente di prestazione

$$\omega = \frac{Q_{IN}}{|W|} = \frac{Q_{IN}}{|Q_{OUT}| - Q_{IN}} =$$

~~dato che $|Q_{OUT}|$~~

$$= \left(\frac{|Q_{OUT}|}{Q_{IN}} - 1 \right)^{-1} \geq 0 \quad \text{dato che } |Q_{OUT}| \geq Q_{IN}$$

Esempio: Ciclo di Carnot invertito (ADCBA)



Adiabatica $A \to D$

$Q_{AD} = 0 \qquad -\Delta U_{AD} = W_{AD} = n c_v (T_H - T_L) > 0$

Isoterma $D \to C$

$\Delta U_{DC} = 0 \qquad W_{DC} = nRT_L \ln\left(\frac{V_C}{V_D}\right) > 0$

$Q_{DC} = W_{DC} = nRT_L \ln\left(\frac{V_C}{V_D}\right) > 0 \to Q_{IN}$

Adiabatica $C \to B$

$Q_{CB} = 0 \qquad W_{CB} = -\Delta U_{CB} = n c_v (T_L - T_H) < 0$

Isoterma $B \to A$

$\Delta U_{BA} = 0 \qquad W_{BA} = nRT_H \ln\left(\frac{V_A}{V_B}\right) < 0 \qquad Q_{BA} = W_{BA} = nRT_H \ln\left(\frac{V_A}{V_B}\right) < 0 \to Q_{OUT}$

$Q_{IN} = nRT_L \ln\left(\frac{V_C}{V_D}\right) \geq 0 \qquad Q_{OUT} = nRT_H \left(\frac{V_A}{V_B}\right) < 0$

Vale sempre che $\frac{V_B}{V_A} = \frac{V_C}{V_D}$

$$\omega = \left(\frac{|Q_{OUT}|}{Q_{IN}} - 1\right)^{-1} = \left(\frac{\cancel{nR}T_H \cancel{\ln\left(\frac{V_B}{V_A}\right)}}{\cancel{nR}T_L \cancel{\ln\left(\frac{V_C}{V_D}\right)}} - 1\right)^{-1} =$$

$$= \left(\frac{T_H}{T_L} - 1\right)^{-1} = \frac{T_L}{T_H - T_L} \xrightarrow{T_L \to 0 \text{ e/o } T_H \to \infty} 0$$

$$\frac{T_L}{T_H - T_L} \xrightarrow{T_H \to T_L} \infty$$ ma attenzione! $W = 0$, $Q_{IN} = |Q_{out}|$

$$|W| = \omega\, Q_{IN} = \frac{T_L}{T_H - T_L}\, Q_{IN}$$

es. $T_H = 20°C$, $T_L = -10°C$ $\rightarrow$ $\omega = 8.8$

$T_H = -10°C$, $T_L = -40°C$ $\rightarrow$ $\omega = 7.8$

La prestazione diminuisce con la temperatura del serbatoio freddo!

Esempio: congelare acqua

$T_L = 0°C = 273.15K$ $\quad T_H \sim 300K$

$$\omega = \frac{273.15}{300 - 273.15} \simeq 10$$

↓

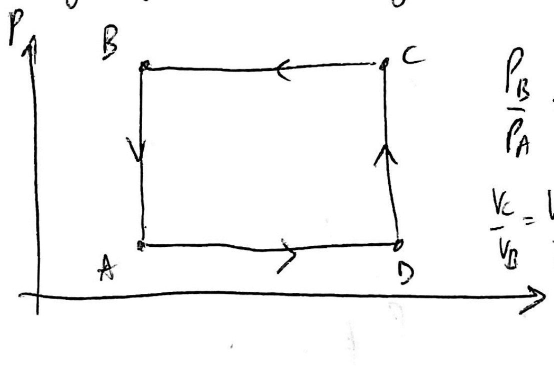
cosa vuol dire?

Ogni J di lavoro esterno convertito in 10 J di calore estratto dalla miscela acqua-ghiaccio

Riassunto:

Ciclo di Carnot	Frigorifero di Carnot
• Q_{IN} assorbito dal serbatoio caldo	• Q_{IN} assorbito da serbatoio freddo
• $\lvert Q_{OUT} \rvert$ ceduto al serbatoio freddo	• $\lvert Q_{out} \rvert$ ceduto a serbatoio caldo
$0 \leq \eta = 1 - \frac{T_L}{T_H} \leq 1$ (si vuole $T_H \gg T_L$)	$\omega = \frac{T_L}{T_H - T_L}$

Frigorifero "rettangolare"



Ricordiamo che come macchina termica

$$\frac{P_B}{P_A} = \frac{P_C}{P_D} = \alpha > 1$$

$$\frac{V_C}{V_B} = \frac{V_D}{V_A} = \beta > 1$$

$$\eta = 1 - \frac{\beta(\alpha-1)+\gamma(\beta-1)}{(\alpha-1)+\gamma\alpha(\beta-1)} \xrightarrow[\gamma=\frac{5}{3}]{\alpha=\beta=2} \frac{2}{13}$$

$(\sim 15\%)$

Come frigorifero

$$\omega = \frac{Q_{IN}}{|W|} = \frac{Q_{IN}}{|Q_{OUT}| - Q_{IN}}$$

$T_D = \beta T_A$
$T_C = \alpha T_D = \alpha\beta T_A$

$$Q_{IN} = Q_{AD} + Q_{BC} = nc_p(T_D - T_A) + nc_v(T_C - T_D) = nc_p T_A(\beta-1) + nc_v T_A \beta(\alpha-1)$$

$$Q_{OUT} = Q_{CB} + Q_{BA} = nc_p(T_B - T_C) + nc_v(T_A - T_B) = nc_p(1-\beta)\alpha T_A + nc_v(1-\alpha)T_A$$

$T_B = \alpha T_A$
$T_C = \beta T_B$

$$|Q_{OUT}| = -Q_{OUT} = nc_p T_A \alpha(\beta-1) + nc_v T_A(\alpha-1)$$

$$W = Q_{IN} - |Q_{OUT}| = nc_p T_A\left[(\beta-1) - \alpha(\beta-1)\right] + nc_v T_A\left[\beta(\alpha-1) - (\alpha-1)\right] =$$

$$= \cancel{n(c_p-c_v)T_A}\; nc_p T_A\left[\beta - 1 - \alpha\beta + \alpha\right] + nc_v T_A(\alpha\beta - \beta - \alpha + 1) =$$

$$= n(c_p - c_v)(\beta - 1 - \alpha\beta + \alpha)T_A \overset{c_p - c_v = R}{=} nRT_A(\beta - 1 - \alpha\beta + \alpha) < 0$$

essendo $\beta - 1 - \alpha\beta + \alpha = (\underset{>0}{\beta-1})(\underset{<0}{1-\alpha})$

$$\omega = \frac{Q_{IN}}{|W|}$$

con $Q_{IN} = nc_p T_A(\beta-1) + nc_v T_A\beta(\alpha-1)$

$$|W| = -W = nRT_A(\alpha\beta - \alpha - \beta + 1) = nRT_A(\underset{>0}{\alpha-1})(\underset{>0}{\beta-1}) > 0$$

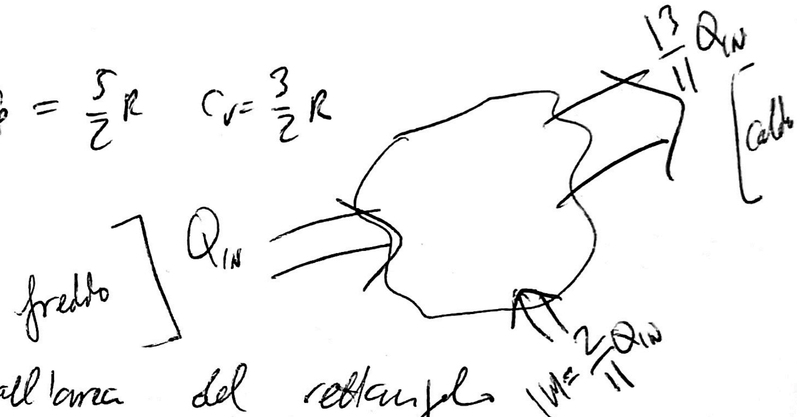
$$\omega = \frac{Q_{IN}}{|W|} = \frac{n c_p T_A (\beta-1) + n c_v T_A \beta(\alpha-1)}{n R T_A (\alpha\beta - \alpha - \beta + 1)} =$$

$$= \frac{\cancel{n} c_p \cancel{T_A}}{\cancel{n} R \cancel{T_A}} \frac{\cancel{(\beta-1)}}{(\alpha-1)\cancel{(\beta-1)}} + \frac{\cancel{n} c_v \cancel{T_A}}{\cancel{n} R \cancel{T_A}} \frac{\beta \cancel{(\alpha-1)}}{\cancel{(\alpha-1)}(\beta-1)} = \frac{c_p}{R}\frac{1}{\alpha-1} + \frac{c_v}{R}\frac{\beta}{\beta-1}$$

Mettiamo dei numeri

$\alpha = \beta = 2$, gas monoatomico $\longrightarrow$ $c_p = \frac{5}{2}R$ $\quad c_v = \frac{3}{2}R$

$$\omega = \frac{5}{2} + \frac{3}{2} \times 2 = \frac{5}{2} + 3 = \frac{11}{2}$$



Nota: W si poteva determinare dall'area del rettangolo

$$|W| = (P_B - P_A)(V_D - V_A) = (\alpha-1)P_A(\beta-1)V_A = (\alpha-1)(\beta-1)P_A V_A \overset{PV=nRT}{=} nRT_A(\alpha-1)(\beta-1) =$$

$$\cancel{= nRT_A(\alpha+\beta-\alpha\beta-1) \Longrightarrow W = -|W| = nRT_A(\alpha\beta+1-\alpha-\beta)}$$

$$= nRT_A(\alpha\beta + 1 - \alpha - \beta) \Rightarrow W = -|W| = nRT_A(\alpha + \beta - \alpha\beta - 1) \quad \checkmark \text{ CVD}$$

Conto alternativo

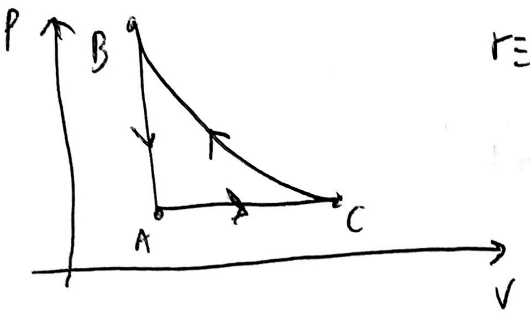
$$\omega = \left(\frac{|Q_{OUT}|}{Q_{IN}} - 1\right)^{-1} = \left(\frac{n c_p T_A \alpha(\beta-1) + n c_v T_A(\alpha-1)}{n c_p T_A(\beta-1) + n c_v T_A \beta(\alpha-1)} - 1\right)^{-1} =$$

$$= \left(\frac{c_p\alpha(\beta-1) + c_v(\alpha-1)}{c_p(\beta-1) + c_v\beta(\alpha-1)} - 1\right)^{-1} = \left(\frac{c_p\alpha(\beta-1) + c_v(\alpha-1) - c_p(\beta-1) - c_v\beta(\alpha-1)}{c_p(\beta-1) + c_v\beta(\alpha-1)}\right)^{-1} =$$

$$= \left(\frac{c_p(\alpha\beta - \alpha + 1 - \beta) + c_v(\alpha + \beta - \alpha\beta - 1)}{c_p(\beta-1) + c_v\beta(\alpha-1)}\right)^{-1} = \left(\frac{(c_p - c_v)[\alpha\beta - \alpha - \beta + 1]}{c_p(\beta-1) + c_v\beta(\alpha-1)}\right)^{-1} = \left(\frac{R(\alpha\beta - \alpha - \beta + 1)}{c_p(\beta-1) + c_v\beta(\alpha-1)}\right)^{-1}$$

$$= \frac{c_p(\beta-1)}{R(\alpha\beta - \alpha - \beta + 1)} + \frac{c_v\beta(\alpha-1)}{R(\alpha\beta - \alpha - \beta + 1)} = \frac{c_p}{R}\frac{1}{\alpha-1} + \frac{c_v}{R}\frac{\beta}{\beta-1} \quad \checkmark \text{ CVD}$$

Frigorifero di Lenoir



$r \equiv \frac{V_C}{V_A}$

$$\omega = \left(\frac{|Q_{OUT}|}{Q_{IN}} - 1\right)^{-1}$$

$$Q_{OUT} = Q_{BA} = nc_v(T_A - T_B) \rightarrow |Q_{OUT}| = nc_v(T_B - T_A)$$

$$Q_{IN} = Q_{AC} = nc_p(T_C - T_A)$$

$$\omega = \left(\frac{|Q_{OUT}|}{Q_{IN}} - 1\right)^{-1} = \left(\frac{\cancel{n}c_v(T_B - T_A)}{\cancel{n}c_p(T_C - T_A)} - 1\right)^{-1} = \left(\frac{1}{\gamma}\frac{(T_B - T_A)}{(T_C - T_A)} - 1\right)^{-1} =$$

$$= \left(\frac{1}{\gamma}\frac{\cancel{T_A}(T_B/T_A - 1)}{\cancel{T_A}(T_C/T_A - 1)} - 1\right)^{-1} = \left(\frac{1}{\gamma}\frac{T_B/T_A - 1}{T_C/T_A - 1} - 1\right)^{-1}$$

$$\frac{T_C}{T_A} = \frac{V_C}{V_A} = r$$

$P_A = P_C$ $\qquad\qquad$ $V_B = V_A$

$$\frac{T_B}{T_A} = \frac{P_B}{P_A} = \frac{P_B}{P_C} \qquad \text{ma} \quad P_B V_B^{\gamma} = P_C V_C^{\gamma} \Rightarrow \frac{P_B}{P_C} = \left(\frac{V_C}{V_B}\right)^{\gamma} = \left(\frac{V_C}{V_A}\right)^{\gamma} = r^{\gamma}$$

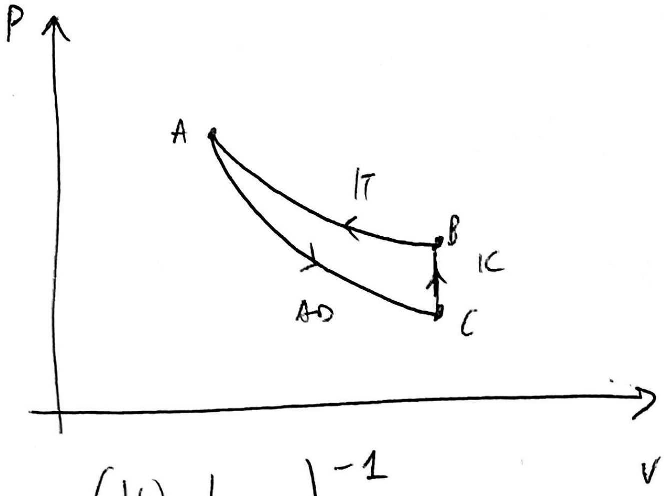
$$\Rightarrow \omega = \left(\frac{1}{\gamma}\frac{r^{\gamma} - 1}{r - 1} - 1\right)^{-1} = \left(\frac{r^{\gamma} - 1 - \gamma r + \gamma}{\gamma(r-1)}\right)^{-1} = \frac{\gamma(r-1)}{r^{\gamma} + \gamma - \gamma r - 1}$$

$$\omega = \frac{\gamma(r-1)}{r^{\gamma} + \gamma - \gamma r - 1}$$

mettiamo di numeri $(\gamma = 5/3)$

- $r = 2$ $\quad \omega \simeq 3.3$
- $r = 10$ $\quad \omega \simeq 0.5$ (non molto efficiente!)

Frigorifero inverso di un ciclo visto prima



$r \equiv \frac{V_B}{V_A} > 1$!

$$\omega = \left(\frac{|Q_{OUT}|}{Q_{IN}} - 1 \right)^{-1}$$

$$Q_{OUT} = Q_{BA} = nRT_A \ln\left(\frac{V_A}{V_B}\right) \Rightarrow |Q_{OUT}| = -Q_{BA} = nRT_A \ln\left(\frac{V_B}{V_A}\right) = nRT_A \ln r$$

$$Q_{IN} = Q_{CB} = nC_V(T_B - T_C) \underset{T_B = T_A}{=} nC_V(T_A - T_C) = \cancel{nC_V T_A\left(\frac{T_C}{T_A} - 1\right)}\ nC_V T_A\left(1 - \frac{T_C}{T_A}\right)$$

$$T_C V_C^{\gamma-1} = T_A V_A^{\gamma-1} \Rightarrow \frac{T_C}{T_A} = \left(\frac{V_A}{V_C}\right)^{\gamma-1} \underset{V_C = V_B}{=} \left(\frac{V_A}{V_B}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} = r^{1-\gamma}$$

$$\Rightarrow \omega = \left(\frac{|Q_{OUT}|}{Q_{IN}} - 1 \right)^{-1} = \left(\frac{nRT_A \ln r}{nC_V T_A (1 - r^{1-\gamma})} - 1 \right)^{-1} = \left(\frac{R \ln r}{C_V (1 - r^{1-\gamma})} - 1 \right)^{-1} =$$

$$= \left(\frac{\ln r}{\frac{C_V}{R}(1 - r^{1-\gamma})} - 1 \right)^{-1} = \left(\frac{\ln r - \frac{C_V}{R}(1 - r^{1-\gamma})}{\frac{C_V}{R}(1 - r^{1-\gamma})} \right)^{-1} = \frac{\frac{C_V}{R}(1 - r^{1-\gamma})}{\ln r - \frac{C_V}{R}(1 - r^{1-\gamma})}$$

$r = 2$ ~4 $(\gamma = \frac{5}{3})$

$r = 10$ ~1.04

poco efficiente!!!

Frigorifero come pompa di calore

$$\omega = \frac{Q_{IN}}{|W|} \Rightarrow |W| = \omega Q_{IN} \quad e \quad |W| = |Q_{OUT}| - Q_{IN} \Rightarrow |Q_{OUT}| = |W| + Q_{IN} = \omega Q_{IN} + Q_{IN} = (\omega + 1) Q_{IN}$$

$|Q_{OUT}| = (\omega + 1) Q_{IN}$

Kelvin (1852): può essere conveniente scaldare una casa (dando Q_{OUT}) raffreddando l'esterno (togliendo Q_{IN}), usato in Scozia dal ~1920, dal 1938 girano "pompe di calore" che possono raffreddare la casa in estate invertendo il verso del gas