

1 Dimostrare che se $\delta W = VdP + PdV$ allora il calore è una funzione di stato

$$\delta W = VdP + PdV = d(PV) \leftarrow \text{differenziale esatto}$$

$$\rightarrow W = \int \delta W = \int d(PV) = PV + \text{const}$$

$\rightarrow W = \text{funzione di stato}$

$$dU = \delta Q - \delta W$$

$$\Rightarrow \delta Q = dU + \delta W = dU + d(PV) = d(U + PV)$$

\uparrow
differenziale esatto!

$$Q = \int \delta Q = U + PV + \text{const} = \boxed{nC_V T + PV + \text{const}}$$

\uparrow
funzione di stato

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Gas ideale, n moli - Trasformazione

$P(V) = \alpha V^2$ $\alpha > 0$ da V_i a $V_f = \beta V_i$ $\beta > 1$

Calcolare:

- ΔU
 - W
 - Q
- $\left\{ \begin{array}{l} \text{dal primo principio} \\ \text{da } \delta Q = n c_p \delta T - V dp \end{array} \right.$

$P_i = \alpha V_i^2 \rightarrow P_f = \alpha V_f^2 = \alpha (\beta V_i)^2 = \beta^2 (\alpha V_i^2) = \beta^2 P_i$

ΔU

$\Delta U = n c_v \Delta T$

$PV = nRT \rightarrow T_i = \frac{P_i V_i}{nR} = \frac{\alpha V_i^3}{nR}$

$P_f = \beta^2 P_i$
 $V_f = \beta V_i$

$T_f = \frac{P_f V_f}{nR} = \frac{\beta^3 P_i V_i}{nR} = \beta^3 T_i$

$T_i = \frac{P_i V_i}{nR}$

$\Delta U = n c_v \Delta T = n c_v (T_f - T_i) = n c_v (\beta^3 T_i - T_i) = n c_v (\beta^3 - 1) T_i$

W

$W = \int_{V_i}^{V_f} P(V) dV = \int_{V_i}^{V_f} \alpha V^2 dV = \frac{\alpha}{3} V^3 \Big|_{V_i}^{V_f} = \frac{\alpha}{3} (V_f^3 - V_i^3) = \frac{\alpha}{3} (\beta^3 V_i^3 - V_i^3) = \frac{\alpha}{3} (\beta^3 - 1) V_i^3$

• Q

Dal primo principio

$$Q = \Delta U + W = nC_v(\beta^3 - 1)T_i + \frac{\alpha}{3}(\beta^3 - 1)V_i^3 = nC_v(\beta^3 - 1)\frac{\alpha V_i^3}{\alpha R} + \frac{\alpha}{3}(\beta^3 - 1)V_i^3 =$$

$$= \alpha C_v(\beta^3 - 1)\frac{V_i^3}{R} + \frac{\alpha(\beta^3 - 1)}{3}V_i^3 = \left(\alpha(\beta^3 - 1)V_i^3 \left[\frac{C_v}{R} + \frac{1}{3} \right] \right)$$

Da $dQ = nC_p dT - PdV$

$$Q = \int_i^f dQ = \int_{T_i}^{T_f} nC_p dT - \int_{P_i}^{P_f} PdV$$

Come scrivere dT e dP ?

$$T = \frac{PV}{nR} = \frac{\alpha V^2 V}{nR} = \frac{\alpha V^3}{nR} \rightarrow dT = d\left(\frac{\alpha V^3}{nR}\right) = \frac{\alpha}{nR} d(V^3) = \frac{3\alpha V^2 dV}{nR}$$

$$P = \alpha V^2 \rightarrow dP = d(\alpha V^2) = \alpha d(V^2) = 2\alpha V dV$$

$$Q = \int dQ = \int dT nC_p - \int dP V = \int dV \frac{3\alpha V^2}{nR} nC_p - \int dV 2\alpha V V =$$

$$= \int dV \left[\frac{3\alpha C_p}{R} V^2 - 2\alpha V^2 \right] = \int dV \left(\frac{3\alpha C_p}{R} - 2\alpha \right) V^2 = \left(\frac{3\alpha C_p}{R} - 2\alpha \right) \int_{V_i}^{V_f} dV V^2 =$$

$$= \left(\frac{3\alpha C_p}{R} - 2\alpha \right) \frac{V^3}{3} \Big|_{V_i}^{V_f} = \alpha \left(\frac{C_p}{R} - \frac{2}{3} \right) [V_f^3 - V_i^3] = \alpha \left(\frac{C_p}{R} - \frac{2}{3} \right) [\beta^3 V_i^3 - V_i^3] =$$

$$= \alpha (\beta^3 - 1) V_i^3 \left[\frac{C_p}{R} - \frac{2}{3} \right]$$

$$= \alpha (\beta^3 - 1) V_i^3 \left[\frac{C_v}{R} + \frac{1}{3} \right] \quad \& \quad \frac{C_p}{R} - \frac{2}{3} = \frac{C_v}{R} + \frac{1}{3}$$

Verifichiamo

$$\frac{C_p}{R} - \frac{2}{3} = \frac{C_v + R}{R} - \frac{2}{3} = \frac{C_v}{R} + 1 - \frac{2}{3} = \frac{C_v}{R} + \frac{1}{3}$$

QED ✓

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DF8, 2

Controllare

numeri alla fine
unità SI
unità fornano

● Rame

$$V_i = 1 \text{ dm}^3$$

$$T_i = 20^\circ\text{C}$$

$$\longrightarrow T_f = 30^\circ\text{C}$$

$$P = P_{atm} = 1 \text{ atm}$$

$$\rho = 8.94 \frac{\text{g}}{\text{cm}^3}$$

$$c_p^{(m)} = 380 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$\beta = 51 \times 10^{-6} \text{ K}^{-1}$$

Calcolare:

- a) 2 fornito, $\Delta U, \Delta H$
- b) Tempo massimo consumo Q fornito tramite resistore elettrico $P = 500 \text{ W}$
- c) Altezza da cui bisogna buttare il blocco affinché calore sviluppato nell'urto tutto assorbito dal blocchetto

a) $H = U + PV$

$$dH = dU + d(PV) = dU + PdV + VdP = \delta Q - PdV + PdV + VdP = \delta Q + VdP$$

Quindi a pressione costante, $dP = 0$, ~~δ~~ $dH = \delta Q$

$$\delta H = Q = m c_p^{(m)} \Delta T = \rho V c_p^{(m)} \Delta T$$

● $V = 1 \text{ dm}^3 = 10^{-3} \text{ m}^3$

$$\Delta T = 10^\circ\text{C} = 10 \text{ K}$$

(buona norma prima trasformare tutte le T in K)

$$\rho = 8.94 \frac{\text{g}}{\text{cm}^3} = 8.94 \frac{10^3 \text{ kg}}{10^{-6} \text{ m}^3} = 8.94 \times 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$Q = \rho V c_p^{(m)} \Delta T = 8.94 \times 10^3 \times 10^{-3} \times 380 \times 10 = 3.4 \times 10^4 \text{ J}$$

= ΔH (dato che $P = \text{const}$)

$$\Delta U = Q - W$$

$$W = P \Delta V = P \beta V \Delta T = 10^5 \times 5.1 \times 10^{-5} \times 10^{-3} \times 10 \text{ Pa} \frac{\text{L}}{\text{K}} \text{ m}^3 \text{ K} = 0.05 \text{ Pa m}^3 = 0.05 \frac{\text{N}}{\text{m}^2} \text{ m}^3 = 0.05 \text{ N} \cdot \text{m} = 0.05 \text{ J} \ll Q$$

$$\Rightarrow \Delta U = Q - W \approx Q \approx 3.4 \times 10^4 \text{ J}$$

$$b) P = \frac{Q}{\Delta t} \rightarrow \Delta t = \frac{Q}{P} = \frac{3.4 \times 10^4 \text{ J}}{500 \text{ W}} \approx 68 \text{ s} \quad (\sim 1 \text{ min})$$

e se invece volessi riscaldare H_2O con lo stesso resistore?

$$c_p = 4186 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

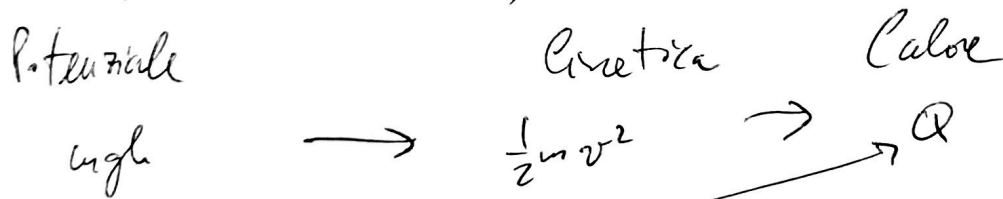
$$\rho = 1 \frac{\text{g}}{\text{cm}^3}$$

$$\Delta t = \frac{Q}{P} = \frac{\rho V c_p \Delta T}{P}$$

$$\Delta t_{H_2O} = \frac{Q_{H_2O}}{P} = \frac{\rho_{H_2O} V c_{p,H_2O} \Delta T}{P} \quad \left(\frac{\rho_{H_2O}}{\rho_{Cu}} \right) \left(\frac{c_{p,H_2O}}{c_{p,Cu}} \right) \Delta t_{Cu} =$$

$$\Delta t_{Cu} = \frac{Q_{Cu}}{P} = \frac{\rho_{Cu} V c_{p,Cu} \Delta T}{P} = \frac{1}{8.94} \frac{4186}{380} t_{Cu} \approx 1.23 t_{Cu} \approx 84 \text{ s} \quad (\sim 1.5 \text{ min})$$

c) Conservazione dell'energia (assumendo tutta energia cinetica assorbita come calore)



$$Q = mgh \rightarrow h = \frac{Q}{mg} = \frac{3.4 \times 10^4 \text{ J}}{8.94 \text{ kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2}} \approx 387 \frac{\text{J}}{\text{N}} = 387 \text{ m}$$

4 $n = 0.4$ mol gas monoatomico ideale

$$V_i = 1 \text{ l}$$

$$P_i = 10 \text{ bar}$$

↓ Espansione adiabatica

$$P_f = 1 \text{ bar}$$

Calcolare $\begin{cases} a) V_f, T_f \\ b) W \end{cases}$

a) $C_v = \frac{3}{2}R$ $C_p = \frac{5}{2}R \rightarrow \gamma = \frac{C_p}{C_v} = \frac{5}{3}$

$P_i V_i^\gamma = P_f V_f^\gamma \Rightarrow V_f = \left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}} V_i = 10^3 \text{ l} \approx 3.98 \text{ l} = 3.98 \times 10^{-3} \text{ m}^3$

$T_f = \frac{P_f V_f}{nR}$ (da $PV = nRT$) $P_f = 1 \text{ bar} = 10^5 \text{ Pa}$

$T_f = \frac{10^5 \times 3.98 \times 10^{-3}}{0.4 \times 8.31} \frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \frac{\text{J}}{\text{K} \cdot \text{mol}}} \approx 119.7 \frac{\text{Pa} \cdot \text{m}^3}{\frac{\text{J}}{\text{K}}} = 119.7 \frac{\text{J}}{\text{K}} = 119.7 \text{ K}$

b) $\Delta U = Q - W \Rightarrow W = Q - \Delta U = -\Delta U = -nC_v \Delta T =$
 $= nC_v (T_i - T_f) = \frac{3}{2} R n (T_i - T_f)$
Q=0 / adiabatica

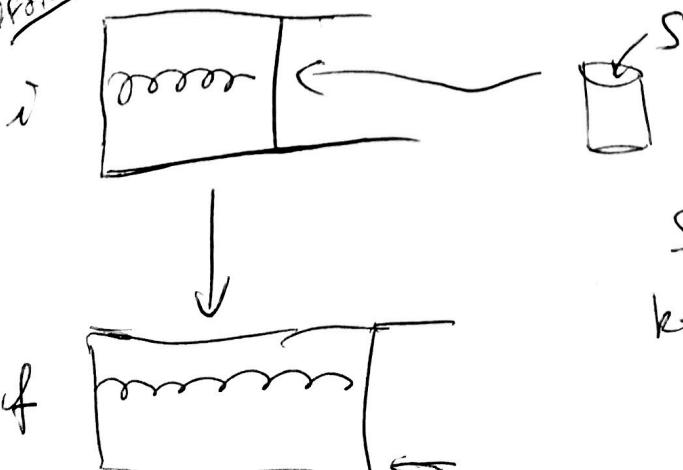
$T_f \approx 119.7 \text{ K}$

$T_i = \frac{P_i V_i}{nR} = \frac{10^6 \text{ Pa} \times 10^{-3} \text{ m}^3}{0.4 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}} \approx 300.8 \text{ K}$

$W = -\Delta U = \frac{3}{2} nR (T_i - T_f) = \frac{3}{2} \times 0.4 \times 8.31 \times (300.8 - 119.7) \frac{\text{mol} \cdot \frac{\text{J}}{\text{K}}}{\text{K} \cdot \text{mol}} \approx 903 \text{ J}$

$\Delta U = -903 \text{ J}$ (gas si raffredda)

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molta a riposo

gas ideale
manometrico

$S = 1 \text{ dm}^2$
 $k = 2 \times 10^4 \frac{\text{N}}{\text{m}}$

$P_i = P_{atm} = 10^5 \text{ Pa}$
 $V_i = 2 \text{ dm}^3$
 $V_f = 1.2 \text{ dm}^3$

$T_f = 300 \text{ K}$

Calcolare $\begin{cases} a) n \\ b) dP/dV \text{ e } P(V) \\ c) P_f, T_f \\ d) Q, W, \Delta U \end{cases}$

a) $PV = nRT$

$$\rightarrow n = \frac{PV}{RT} = \frac{10^5 \times 10^{-3}}{8.31 \times 300} \approx 0.04 \text{ mol}$$

b) $P = P_{atm} + \frac{F}{S} = P_{atm} + \frac{kx}{S} \implies dP = d(P_{atm} + \frac{kx}{S}) = d(\frac{kx}{S}) = \frac{k}{S} dx$
legge di Hooke

$$V = S \cdot x \rightarrow dV = d(Sx) = S dx$$

$$\rightarrow \frac{dP}{dV} = \frac{\frac{k}{S} dx}{S dx} = \frac{k}{S^2} \quad dP = \frac{k}{S^2} dV \text{ relazione lineare}$$

c) Lunghezza iniziale $1 \text{ dm} = 10^{-2} \text{ m}$
 Lunghezza finale $1.2 \text{ dm} = 1.2 \times 10^{-2} \text{ m}$ } dato $V_i = 1 \text{ dm}^3$
 due $V_f = 1.2 \text{ dm}^3$
 $U = S \cdot l \quad S = 1 \text{ dm}^2$

$$x = 0.2 \text{ dm} = 2 \times 10^{-2} \text{ m}$$

$$\Delta V = 0.2 \text{ dm}^3 = 0.2 \times 10^{-3} \text{ m}^3 = 2 \times 10^{-4} \text{ m}^3$$

$$S = 1 \text{ dm}^2 = 10^{-2} \text{ m}^2$$

$$P_f = P_i + \int dV \frac{dP}{dV} = P_{atm} + \int dV \frac{k}{S^2} = P_{atm} + \frac{k}{S^2} \Delta V = 10^5 \text{ Pa} + \frac{2 \times 10^4 \times 2 \times 10^4}{(10^{-2})^2} \text{ Pa} = 1.4 \times 10^5 \text{ Pa}$$

$$PV = nRT$$

$$\rightarrow T_f = \frac{P_f V_f}{nR} = \frac{1.4 \times 10^5 \times 1.2 \times 10^{-3}}{0.04 \times 8.31} \text{ K} \approx 505 \text{ K}$$

$$d) \Delta U = n c_v \Delta T = \frac{3}{2} n R \Delta T = \frac{3}{2} n R (T_f - T_i) = \frac{3}{2} \times 0.06 \times 8.31 \times (500 - 300) \approx 102 \text{ J}$$

$c_v = \frac{3}{2} R$

$$dW = P dV$$

$$\rightarrow W = \int dV P(V) = \int dV [P_{atm} + P_{molla}] = P_{atm} \int dV + \int dV \frac{kx}{s} =$$

$$= P_{atm} \int dV + \int dx \frac{k}{s} x = P_{atm} \frac{\Delta V}{20 \text{ dm}^3} + \int_0^{x_f} dx kx =$$

$$= P_{atm} \Delta V + \frac{kx^2}{2} \Big|_0^{x_f} = P_{atm} \Delta V + k \frac{x_f^2}{2}$$

$$x_f = 0.2 \text{ dm} = 2 \times 10^{-2} \text{ m}$$

$$W = 10^5 \text{ Pa} \times 2 \times 10^{-4} \text{ m}^3 + 2 \times 10^4 \frac{\text{N}}{\text{m}} \times \frac{(2 \times 10^{-2} \text{ m})^2}{2} = 24 \text{ J}$$

$$\Delta U = Q - W$$

$$\Rightarrow Q = \Delta U + W \approx 102 \text{ J} + 24 \text{ J} = 126 \text{ J}$$

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Cilindro con pareti diaterme e verticale

${}^4\text{He}$ $m = 8 \text{ g}$ $T = 27^\circ\text{C}$

a) Espansione isobara fino a $V \rightarrow 2V$. Calcolare W , Q , ΔU , ΔH

Assumiamo $P = P_{atm} = 10^5 \text{ Pa}$ (se non è esplicitamente detto)

$$T_i = 27^\circ\text{C} \approx 300 \text{ K}$$

$$P_i = P_{atm} = 10^5 \text{ Pa}$$

$$n = \frac{\text{massa totale}}{\text{massa molare}} = \frac{8 \text{ g}}{4 \text{ g/mol}} = 2 \text{ mol}$$

$$V_i = \frac{n R T_i}{P_i} = \frac{2 \times 8.31 \times 300}{10^5} \text{ m}^3 \approx 4.986 \times 10^{-2} \text{ m}^3$$

Stato iniziale

$$P_f = P_i = 10^5 \text{ Pa}$$

$$V_f = 2V_i = 9.972 \times 10^{-2} \text{ m}^3$$

$$T_2 = \frac{P_f V_f}{nR} = \frac{2P_i V_i}{nR} = 2T_i = 600 \text{ K}$$

$$n = 2 \text{ mol}$$

Stato finale

~~DETERMINA~~

$$W = P \Delta V = P(V_2 - V_1) = 10^5 \times (9.972 - 4.986) \times 10^{-2} = 4986 \text{ J} > 0$$

gas fa lavoro

$$Q = n c_p \Delta T = \frac{5}{2} n R \Delta T = \frac{5}{2} \times 2 \times 8.31 \times (600 - 300) = 12465 \text{ J}$$

\uparrow
He monoatomico

$$\Delta U = Q - W = 12465 \text{ J} - 4986 \text{ J} = 7479 \text{ J}$$

$$n c_v \Delta T = \frac{3}{2} n R \Delta T = \frac{3}{2} \times 2 \times 8.31 \times (600 - 300) = 7479 \text{ J}$$

$$H = U + PV$$

Sappiamo già che dovremmo trovare $\Delta H = Q_p = Q = 12465 \text{ J}$

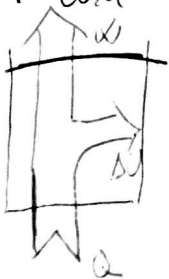
$$P_i V_i = 10^5 \text{ Pa} \times 4.986 \times 10^{-2} \text{ m}^3 = 4986 \text{ J}$$

$$P_f V_f = 10^5 \text{ Pa} \times 9.972 \times 10^{-2} \text{ m}^3 = 9972 \text{ J}$$

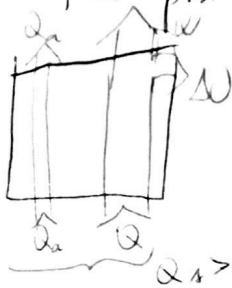
$$\Delta(PV) = P_f V_f - P_i V_i = 9972 \text{ J} - 4986 \text{ J} = 4986 \text{ J}$$

$$\Delta H = \Delta U + \Delta(PV) = 7479 \text{ J} + 4986 \text{ J} = 12465 \text{ J} = Q!$$

b) Come succede e che attrito fra pistone e cilindro?



Senza attrito



Con attrito

$$Q_1 > Q$$

Con attrito da a parità di lavoro utile PSV

- il gas deve fare più lavoro
- la trasformazione non è reversibile
- il gas deve assorbire più calore

F : risultante delle forze d'attrito

S : sezione cilindro

$W_{tot} = W + W_a = (P + \frac{F}{S}) \Delta V$ di cui

- lavoro utile PSV
- dissipato in calore $\frac{F}{S} \Delta V$

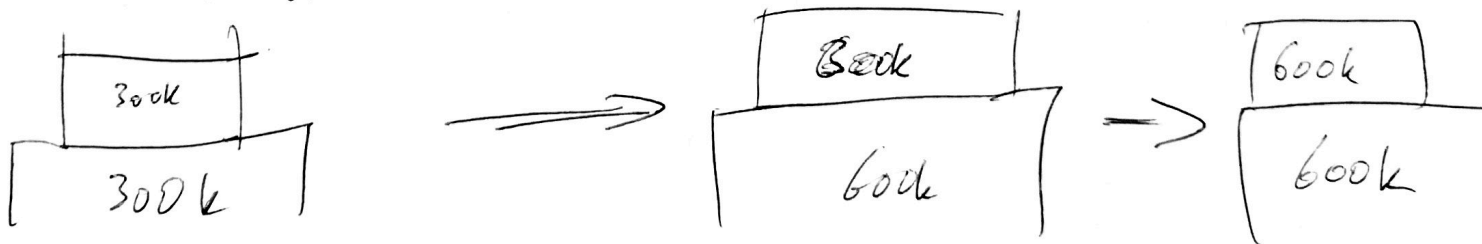
$Q_{tot} = \Delta U + W_{tot} = \Delta U + \underbrace{W + W_a}_Q = Q + W_a$

Quindi il sistema deve assorbire Q_a in più, che poi perde calore tutto all'ambiente!

Quantità netta di calore ceduta dall'ambiente: $Q + Q_a - Q_a = Q$ come prima!

c) Cilindro spostato rapidamente da $T_i = 300K$ a $T_f = 600K$
 $P_i = 1 \text{ atm} \rightarrow P_f = 1 \text{ atm}$

Come cambiano i risultati?



Moto del pistone: accelerazione eterna, poi oscillazioni intorno alla posizione di equilibrio



stesso stato finale del caso precedente

$W = P \Delta V = 49865$
 $\Delta U = 74795$
 $Q = 124655$