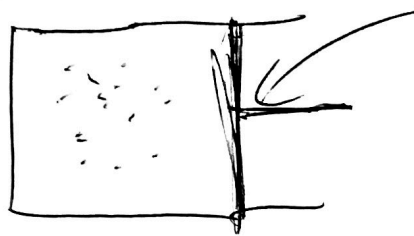


7

8,3



pistone
senza
attrito

$n = 6 \text{ mol}$

$P_i = 6 \text{ bar} = 6 \times 10^5 \text{ Pa}$

T_i

isoterma reversibile

$P_f = 2 \text{ bar} = 2 \times 10^5 \text{ Pa}$

$Q = 12000 \text{ J}$

Cilindro diatermico, orizzontale

a) Calcolare T_i, V_i, V_f

Isoterma $\Delta U = 0 = Q - W$

$\Rightarrow Q = W$

$PV = nRT \rightarrow PV = \text{cost}$

$W = \int P dV = \int P dV \frac{nRT}{V} = nRT_i \int \frac{dV}{V} = nRT_i \ln\left(\frac{V_f}{V_i}\right) = nRT_i \ln\left(\frac{P_i}{P_f}\right)$

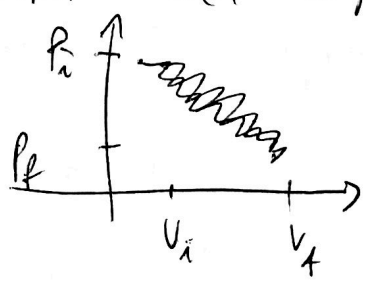
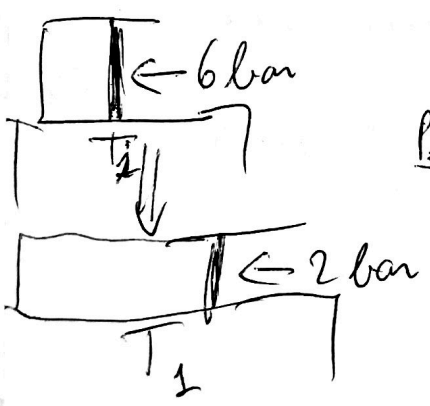
$Q = W = nRT_i \ln\left(\frac{P_i}{P_f}\right) \rightarrow T_i = \frac{Q}{nR \ln(P_i/P_f)} = \frac{Q}{nR \ln 3}$

$= \frac{12000 \text{ J}}{6 \times 8,31 \times \ln 3} \approx 219 \text{ K}$

$V_i = \frac{nRT_i}{P_i} = \frac{6 \times 8,31 \times 219}{6 \times 10^5} \text{ m}^3 \approx 0,018 \text{ m}^3$

$V_f = \frac{nRT_f}{P_f} = \frac{nRT_i}{P_f} = \frac{6 \times 8,31 \times 219}{2 \times 10^5} \text{ m}^3 \approx 0,055 \text{ m}^3$

b) Cilindro posto istantaneamente da $P_i = 6 \text{ bar}$ a $P_f = 2 \text{ bar}$, ambiente tenuto alla temperatura T_i . Calcolare Q, W



trasformazione irreversibile per
pi stati di prova

$\Delta U = 0 \quad (T_i = T_f)$

$W = P_f(V_f - V_i) = 2 \times 10^5 (0,055 - 0,018) \approx 7400 \text{ J}$
 $Q = W = 7400 \text{ J}$

< ~~reversibile~~
 < ~~Q, W~~
 < reversibile

8 DFBK

Cilindro con pareti adiabatiche orizzontale



$V_i = 1 \text{ m}^3 \quad \text{H}_2$

$T_i = 300 \text{ K}$

$P_i = 2 \times 10^5 \text{ Pa}$

a) Espansione reversibile $P_i \rightarrow P_f = \frac{P_i}{2}$ - Calcolare V_f, T_f

$P_f = \frac{P_i}{2} = 10^5 \text{ Pa}$

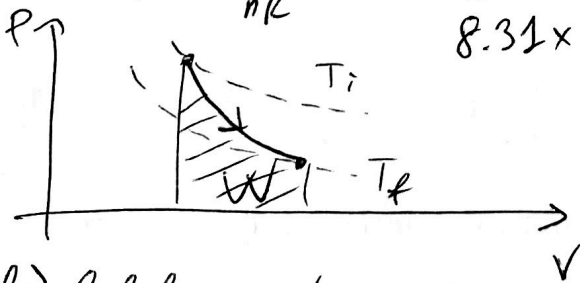
$n = \frac{P_i V_i}{RT} = \frac{2 \times 10^5 \times 1}{8.31 \times 300} \approx 80.2 \text{ mol}$

Trasformazione adiabatica $\rightarrow PV^\gamma = \text{const} \quad \gamma = \frac{C_p}{C_v} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} \Rightarrow \gamma = 1.4$

$P_i V_i^\gamma = P_f V_f^\gamma \rightarrow V_f = \left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}} V_i \approx \frac{2^{5/7}}{1} V_i$

$= 2^{5/7} V_i \approx 1.64 \text{ m}^3$

$T_f = \frac{P_f V_f}{nR} = \frac{10^5 \times 1.64}{8.31 \times 80.2} \approx 246 \text{ K} \rightarrow$ giustamente il gas si raffredda



b) Calcolare W

$\Delta U = Q - W \stackrel{Q=0}{=} -W \Rightarrow W = -\Delta U = -n C_v (T_f - T_i) = n C_v (T_i - T_f) > 0$

$C_v = \frac{5}{2}R \Rightarrow W = \frac{5}{2} n R (T_i - T_f) = \frac{5}{2} \times 80.2 \times 8.31 \times (300 - 246) \text{ J} \approx 9 \times 10^4 \text{ J}$
(ovviamente $\Delta U < 0$)

Possiamo anche calcolare W integrando PdV

$$W = \int dV P(V)$$

$$P_i V_i^\gamma = P V^\gamma \rightarrow P(V) = \frac{P_i V_i^\gamma}{V^\gamma}$$

$$W = \int dV \frac{P_i V_i^\gamma}{V^\gamma} = P_i V_i^\gamma \int_{V_i}^{V_f} dV V^{-\gamma} = \frac{P_i V_i^\gamma}{1-\gamma} \left[V^{1-\gamma} \right]_{V_i}^{V_f} =$$

$$= \frac{P_i V_i^\gamma}{1-\gamma} (V_f^{1-\gamma} - V_i^{1-\gamma}) = -\frac{C_v}{R} P_i V_i^\gamma (V_f^{1-\gamma} - V_i^{1-\gamma}) =$$

$$1-\gamma = 1 - \frac{C_p}{C_v} = \frac{C_v - C_p}{C_v} = -\frac{R}{C_v}$$

$$\frac{1}{1-\gamma} = -\frac{C_v}{R}$$

$$= \frac{C_v}{R} P_i V_i^\gamma (V_i^{1-\gamma} - V_f^{1-\gamma}) =$$

$$= \frac{C_v}{R} (P_i V_i^\gamma V_i^{1-\gamma} - P_i V_i^\gamma V_f^{1-\gamma}) = \frac{C_v}{R} (P_i V_i - P_i V_i^\gamma V_f^{1-\gamma}) =$$

$$= \frac{5}{2} (2 \times 10^5 \times 1 - 2 \times 10^5 \times 1^{\frac{7}{5}} \times 1.64^{-\frac{2}{5}}) \text{ J} \approx 9 \times 10^4 \text{ J} \quad \checkmark$$

c) Cilindro posto da $P_i = 2 \times 10^5 \text{ Pa}$ istantaneamente a $P_f = 10^5 \text{ Pa}$ e lasciato raggiungere l'equilibrio termodinamico. Si può calcolare W ?

Trasformazione irreversibile

Moto del pistone: accelerazione esterna, poi oscillazioni intorno al punto di equilibrio

$$W = P_{\text{ext}} (V_2' - V_1) \quad \text{ma } V_2' \text{ è indeterminato}$$

Mancano informazioni sulla temperatura finale

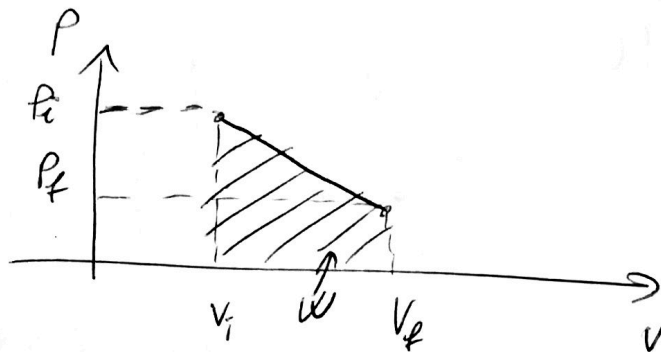
Quindi W non si può calcolare

9 ^{0,885} 400 mol di ${}^4\text{He}$

$P_i = 2 \times 10^5 \text{ Pa}$ $V_i = 6 \text{ m}^3$

variazione
p-v lineare

$P_f = 10^5 \text{ Pa}$ $V_f = 10 \text{ m}^3$



Calcolare Q

$T_i = \frac{P_i V_i}{nR} = \frac{2 \times 10^5 \times 6}{4 \times 10^2 \times 8.31} \approx 361 \text{ K}$

$T_f = \frac{P_f V_f}{nR} = \frac{10^5 \times 10}{4 \times 10^2 \times 8.31} \approx 301 \text{ K}$

● $Q = \Delta U + W$ dal I principio

$\Delta U = n c_v \Delta T = \frac{3}{2} n R \Delta T = \frac{3}{2} n R (T_f - T_i) = \frac{3}{2} \times 400 \times 8.31 \times (301 - 361) \text{ J} \approx -2.99 \times 10^5 \text{ J}$

$W = \int_{V_i}^{V_f} p \, dV$

Determiniamo $p(V)$: $p = \alpha V + \beta$

$\alpha = \frac{\Delta p}{\Delta V} = \frac{P_f - P_i}{V_f - V_i} = -\frac{10^5 \text{ Pa}}{4 \text{ m}^3}$

$P_i = \alpha V_i + \beta \rightarrow \beta = P_i - \alpha V_i =$

$= 2 \times 10^5 \text{ Pa} + 2.5 \times 10^4 \frac{\text{Pa}}{\text{m}^3} \times 6 \text{ m}^3 =$

~~$= 5 \times 10^5 \text{ Pa}$~~ $= 3.5 \times 10^5 \text{ Pa}$

$= -2.5 \times 10^4 \frac{\text{Pa}}{\text{m}^3}$

Quindi la trasformazione è scrivibile: ~~$p(V) = -2.5 \times 10^4 \frac{\text{Pa}}{\text{m}^3} V + 5 \times 10^5 \text{ Pa}$~~

$p(V) = -2.5 \times 10^4 \frac{\text{Pa}}{\text{m}^3} V + 3.5 \times 10^5 \text{ Pa} = \alpha V + \beta$

$\begin{cases} \alpha = -2.5 \times 10^4 \frac{\text{Pa}}{\text{m}^3} \\ \beta = 3.5 \times 10^5 \text{ Pa} \end{cases}$

$W = \int_{V_i}^{V_f} p \, dV = \int_{V_i}^{V_f} (\alpha V + \beta) \, dV = \int_{V_i}^{V_f} \alpha V \, dV + \int_{V_i}^{V_f} \beta \, dV = \frac{\alpha V^2}{2} \Big|_{V_i}^{V_f} + \beta V \Big|_{V_i}^{V_f} =$

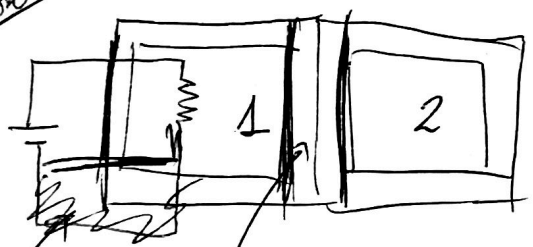
$= \frac{\alpha}{2} (V_f^2 - V_i^2) + \beta (V_f - V_i) = 6 \times 10^5 \text{ J}$ (alternativamente calcolavamo l'area del trapezoido)

$Q = \Delta U + W = 3.01 \times 10^5 \text{ J}$

$W = \frac{(P_f + P_i) \Delta V}{2} = \frac{(10^5 + 2 \times 10^5) (10 - 6)}{2} = 6 \times 10^5 \text{ J}!$

10

0786



Cilindro a pareti
adiabatiche

resistore
elettrico

pistone adiabatico
scorrevole, no attrit

$n_1 = 1 \text{ mol (He)}$ $T_1 = 300 \text{ K}$ $V_{tot} = 3 \text{ l}$
 $n_2 = 2 \text{ mol (He)}$ $T_2 = 300 \text{ K}$

$Q_1 = 1000 \text{ J}$ mediante resistore elettrico in maniera reversibile

Determinare stato finale del gas

Determinare prima lo stato iniziale

↳ sistema all'equilibrio quindi c'è anche equilibrio meccanico

$\hookrightarrow P_1 = P_2$ $T_1 = T_2$

~~$PV = nRT \rightarrow V = \frac{nRT}{P}$~~

~~$V_1 = \frac{n_1 RT}{P}$~~

~~$V_2 = \frac{n_2 RT}{P}$~~

$PV = nRT \rightarrow V = \frac{nRT}{P}$

data $T_1 = T_2$

$V_1 = \frac{n_1 RT}{P}$

$V_2 = \frac{n_2 RT}{P}$

$\left. \begin{aligned} \frac{V_1}{V_2} &= \frac{n_1}{n_2} = \frac{1}{2} \\ V_1 + V_2 &= 3 \text{ l} \end{aligned} \right\} \Rightarrow \begin{aligned} V_1 &= 1 \text{ l} \\ V_2 &= 2 \text{ l} \end{aligned}$

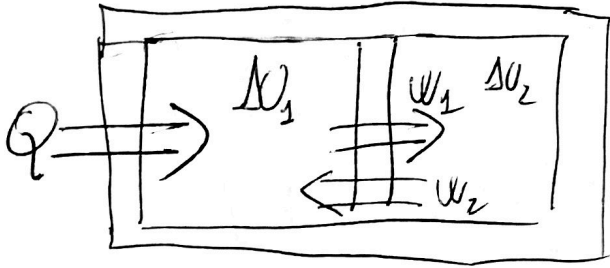
equilibrio
meccanico (data)

Quindi

$\left\{ \begin{aligned} n_1 &= 1 \text{ mol} & V_1 &= 10^{-3} \text{ m}^3 & T_1 &= 300 \text{ K} \\ n_2 &= 2 \text{ mol} & V_2 &= 2 \times 10^{-3} \text{ m}^3 & T_2 &= 300 \text{ K} \end{aligned} \right. \rightarrow$

$P = \frac{nRT}{V} = 24.9 \text{ bar} = 24.9 \times 10^5 \text{ Pa}$

Come procede la trasformazione?



Zona 1 riceve calore dal
 pistone dentro, ma zona 2 non
 ne può ricevere...

Dal III principio di Newton $W_1 = -W_2$

I principio della termodinamica

$$\Delta U = Q - W \Rightarrow Q = \Delta U + W$$

$$\begin{cases} \text{Zona 1: } Q = \Delta U_1 + W_1 \\ \text{Zona 2: } 0 = \Delta U_2 + W_2 = \Delta U_2 - W_1 \end{cases}$$

Sommando

$$Q = \Delta U_1 + \Delta U_2$$

$$Q = \Delta U_1 + \Delta U_2 = n_1 c_v (T_1' - T) + n_2 c_v (T_2' - T)$$

dove T_1' e T_2' sono le nuove temperature delle zone una volta che
 si raggiunge l'equilibrio ($P_1' = P_2' = P'$)

$$\begin{cases} P' V_1' = n_1 R T_1' \\ P' V_2' = n_2 R T_2' \\ V_1' + V_2' = V_1 + V_2 = V_{tot} \end{cases}$$

$P' V_2'^{\gamma} = P V_2^{\gamma}$ dato che in zona 2 la
 trasformazione è adiabatica

5 equazioni 5 incognite ($P', V_1', V_2', T_1', T_2'$)

$$P' V_1' = n_1 R T_1'$$

$$P' V_2' = n_2 R T_2'$$

$$P' (V_1' + V_2') = (n_1 R T_1' + n_2 R T_2') \Rightarrow P' V_{tot} = R (n_1 T_1' + n_2 T_2')$$

$$(n_1 T_1' + n_2 T_2') = \frac{P' V_{tot}}{R}$$

Ma sappiamo

$$Q = n_1 c_v (T_1' - T) + n_2 c_v (T_2' - T) = c_v (n_1 T_1' + n_2 T_2') - c_v (n_1 + n_2) T =$$

$$= \frac{C_v}{R} P' V_{tot} - C_v (n_1 + n_2) T = \frac{3}{2} P' V_{tot} - C_v (n_1 + n_2) T = Q$$

$\uparrow C_v = \frac{3}{2} R$
 \uparrow ~~not~~
not

$$\Rightarrow \frac{3}{2} P' = \frac{Q + C_v (n_1 + n_2) T}{\frac{3}{2} V_{tot}} = 2 \frac{[Q + C_v (n_1 + n_2) T]}{3 V_{tot}} =$$

$$= 2 \frac{[1000 + \frac{3}{2} \times 8.31 \times (1+2) \times 300]}{3 \times 3 \times 10^{-3}} \approx 27.1 \times 10^6 \text{ Pa} = 27.1 \text{ bar}$$

~~27.1 bar = 27.1 \times 10^6 Pa~~
 27.1 bar = 27.1 \times 10^5 Pa

Ora uniamo

$$P' V_2'^{\gamma} = P V_2^{\gamma} \quad \gamma = \frac{5}{3}$$

$$\Rightarrow V_2' = \left(\frac{P}{P'} \right)^{\frac{1}{\gamma}} V_2 = \left(\frac{P}{P'} \right)^{\frac{3}{5}} V_2 = \left(\frac{24.9 \text{ bar}}{27.1 \text{ bar}} \right)^{\frac{3}{5}} V_2 \approx 1.84 \text{ l}$$

$$= 1.84 \times 10^{-3} \text{ m}^3$$

$$V_1' = V_{tot} - V_2' = 3 \times 10^{-3} \text{ m}^3 - 1.84 \times 10^{-3} \text{ m}^3 = 1.16 \times 10^{-3} \text{ m}^3$$

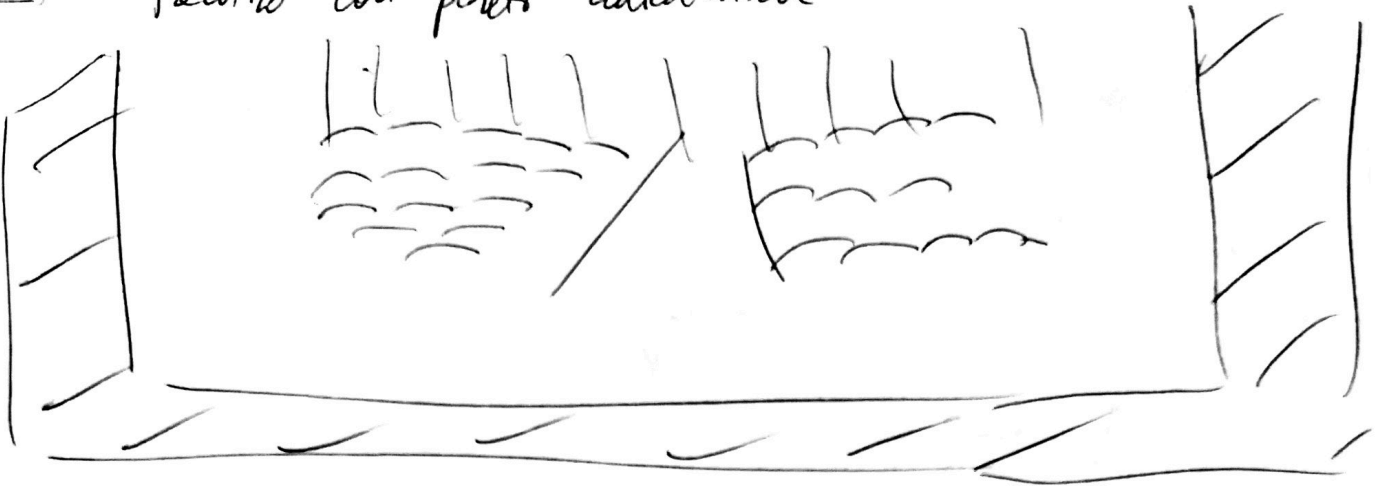
$$PV = nRT \rightarrow T = \frac{PV}{nR}$$

$$\left\{ \begin{aligned} T_1' &= \frac{P_1' V_1'}{nR} = \frac{27.1 \times 10^5 \times 1.16 \times 10^{-3}}{1 \times 8.31} \approx 378 \text{ K} \\ T_2' &= \frac{P_2' V_2'}{nR} = \frac{27.1 \times 10^5 \times 1.84 \times 10^{-3}}{2 \times 8.31} \approx 300 \text{ K} \end{aligned} \right.$$

↳ Determina $P', V_1', V_2', T_1', T_2'$

11

Teatro con pareti adiabatiche



1800 persone, ciascuna trasporta 100 W all'ambiente a riposo

Impianto di condizionamento si ferma

a) Calcolare ΔU dopo 15 minuti

$$\Delta U = Q - W$$

$W \neq 0$ perché le pareti di un teatro sono (si spera) rigide e indeformabili

$$\rightarrow W \approx 0$$

$$\rightarrow \Delta U = Q$$

$$P = \frac{Q}{\Delta t} \rightarrow Q = P \Delta t = 100 \text{ W} \times 900 \text{ s} = 9 \times 10^4 \text{ J per persona}$$

$$Q_{\text{tot}} = 1800 \times 9 \times 10^4 \text{ J} = 1.62 \times 10^8 \text{ J}$$

b) Assumendo $P = 1 \text{ atm}$, $V = (49 \times 35 \times 21) \text{ m}^3$, $T_i = 300 \text{ K}$, $T_f = 300 \text{ K}$, calcolare ΔT

(gas monoatomico N_2)

↑
Scala

$$V = 37730 \text{ m}^3$$

$$n = \frac{PV}{RT} = \frac{10^5 \times 3.773 \times 10^4}{8.31 \times 3 \times 10^2} \approx 1.5 \times 10^6$$

$$\Delta U = n c_v \Delta T \rightarrow \Delta T = \frac{\Delta U}{n c_v} = \frac{\Delta U}{\frac{5}{2} n R} = \frac{1.62 \times 10^8 \text{ J}}{\frac{5}{2} \times 1.5 \times 10^6 \times 8.31} \text{ K} \approx 5.2 \text{ K}$$

c) Calcolare ΔU totale (persone + teatro)

$$\Delta U = Q - W = 0 - 0 = 0 \quad \checkmark \text{ delle persone trasferita all'aria}$$