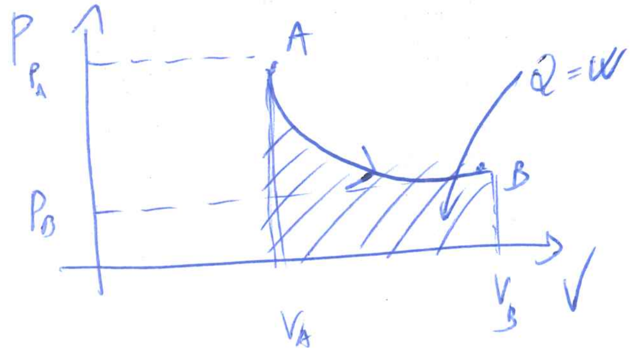
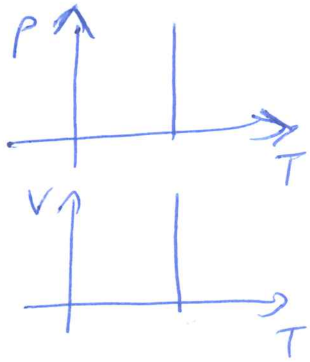
Trasformazione isoterma quasistatica

$T = \text{const}$ $\Rightarrow$ contatto con un termostato $C_T \gg C_{gas}$

espandere o contrarre il gas lentamente (es. cilindro con pistone)

$PV = nRT \longrightarrow$ ramo d'iperbole in P-V





$U = U(T) \Rightarrow \Delta U = U(B) - U(A) = 0 \Rightarrow Q = W$

Gas espanso $(V_B > V_A) \Rightarrow W > 0$ calore passa dal termostato al gas

Gas contratto $(V_B < V_A) \rightarrow W < 0$ calore passa dal gas al termostato

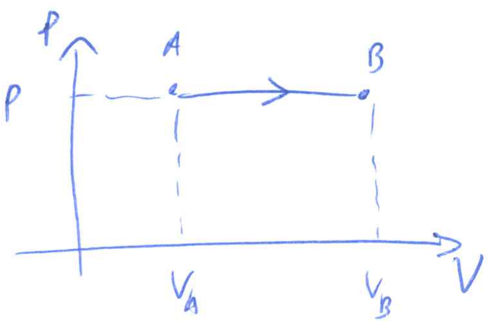
$$\delta W = P dV \rightarrow W = \int_{V_A}^{V_B} dV\, P(V) = \int_{V_A}^{V_B} \frac{nRT}{V} dV = nRT \int_{V_A}^{V_B} \frac{dV}{V} =$$

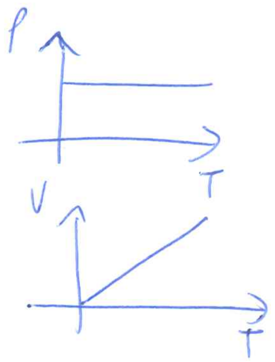
$$= nRT \ln\left(\frac{V_B}{V_A}\right) \qquad W = nRT \ln\left(\frac{V_B}{V_A}\right) \qquad Q = nRT \ln\left(\frac{V_B}{V_A}\right)$$

$|Q, W|$: area sotto la curva P-V

Trasformazione isobara quasistatica

$P = \text{const}$ $\Rightarrow$ a contatto con termostato con temperatura regolabile





$$W = \int_{V_A}^{V_B} dV\, P(V) = P \int_{V_A}^{V_B} dV = P(V_B - V_A) = P\Delta V$$

$$\delta Q = n c_p dT \rightarrow Q = \int_{T_A}^{T_B} n c_p \, dT = n c_p \Delta T$$

$$dU = n c_v dT \rightarrow U = \int_{T_A}^{T_B} n c_v \, dT = n c_v \Delta T$$

Verifichiamo che il primo principio sia soddisfatto

$$W = P\Delta V \qquad Q = nc_p\Delta T \qquad \Delta U = nc_v\Delta T$$

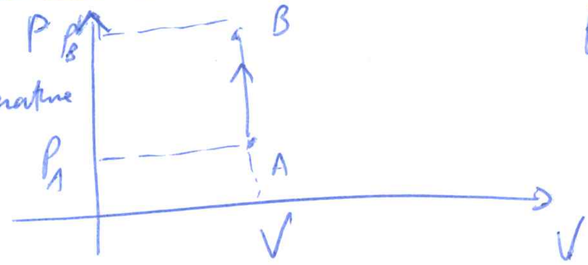
$$Q - W = nc_p\Delta T - P\Delta V = nc_p\Delta T - PV_B + PV_A = nc_p(T_B - T_A) - nRT_B + nRT_A =$$

$$= nc_p(T_B - T_A) - nR(T_B - T_A) = n(c_p - R)(T_B - T_A) = nc_v(T_B - T_A) = nc_v\Delta T = \Delta U$$

$c_p = c_v + R$

Trasformazione isocora quasistatica

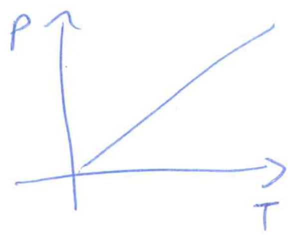
$\Delta V = 0 \Rightarrow$ contatto con termostato con temperatura regolabile

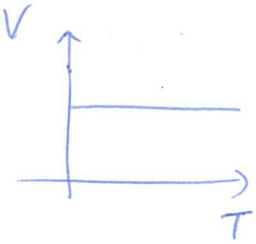


$PV = nRT \rightarrow T = \frac{PV}{nR}$

$$W = P\Delta V = 0 \qquad Q = nc_v(T_B - T_A) = \cancel{n}c_v\left(\frac{P_B V}{nR} - \frac{P_A V}{nR}\right) =$$

$$= \frac{\cancel{n}V}{\cancel{n}R}c_v(P_B - P_A) = \frac{V}{R}c_v\Delta P$$





$$Q = \Delta U = nc_v\Delta T$$

Trasformazione adiabatica quasistatica

$$Q = 0 \qquad \Delta U = Q - W = -W = nc_v\Delta T$$

Usiamo
$$\left.\begin{array}{l} \delta Q = nc_v dT + PdV = 0 \\ \delta Q = nc_p dT - VdP = 0 \end{array}\right\} \longrightarrow nc_v dT + PdV = nc_p dT - VdP = 0$$

$$\begin{cases} nc_v dT = -PdV \\ nc_p dT = VdP \end{cases} \longrightarrow \frac{VdP}{\cancel{n}c_p \cancel{dT}} = -\frac{PdV}{\cancel{n}c_v\cancel{dT}} \Longrightarrow \frac{dP}{P} = -\frac{c_p}{c_v}\frac{dV}{V}$$

$$\gamma \equiv \frac{c_p}{c_v} > 1 \qquad \int\frac{dP}{P} = -\gamma\int\frac{dV}{V} \qquad \ln P = -\gamma\ln V + \text{const} \rightarrow \ln P = -\ln V^\gamma + \text{const}$$

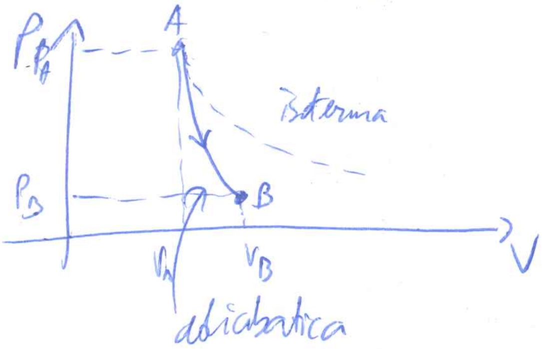
$$\Rightarrow \boxed{\ln PV^\gamma = \text{const}}$$

$\gamma = \frac{5}{3}$ gas monoatomico

$\gamma = \frac{7}{5}$ gas biatomico

$$\boxed{PV^\gamma = const}$$

più ripida di iperbole
dato che $\gamma > 1$



A parità di volume finale, per adiabatica gas a pressione più bassa e quindi a temperatura più bassa

$$W = -nc_v \Delta T \overset{?}{=} \int dV P \qquad PV^\gamma = const \rightarrow P = const\, V^{-\gamma} = \alpha V^{-\gamma}$$

Verifichiamo

$$W = \int_{V_A}^{V_B} dV P(V) = \int_{V_A}^{V_B} dV \alpha V^{-\gamma} = \frac{\alpha}{1-\gamma}\left(V_B^{1-\gamma} - V_A^{1-\gamma}\right) = *$$

Ma $\quad P_B V_B^\gamma = P_A V_A^\gamma = \alpha \implies$

$$\alpha V_B^{1-\gamma} = P_B V_B$$
$$\alpha V_A^{1-\gamma} = P_A V_A$$

$$* = \frac{P_B V_B - P_A V_A}{1-\gamma} = -\frac{c_v}{R}\left(P_B V_B - P_A V_A\right)$$

$$\gamma = \frac{c_p}{c_v} = \frac{c_v + R}{c_v} = 1 + \frac{R}{c_v} \Rightarrow 1-\gamma = -\frac{R}{c_v}$$

$$W = -\frac{c_v}{R}\left(P_B V_B - P_A V_A\right) = -\frac{c_v n \cancel{R}}{\cancel{R}}\left(T_B - T_A\right) = -nc_v \Delta T$$

Esprimiamo $PV^\gamma = const$ in termini di T, V

$$const = PV^\gamma = \frac{nRT}{V} V^\gamma = nRTV^{\gamma-1} = const \longrightarrow \boxed{TV^{\gamma-1} = const}$$

$$const = PV^\gamma = P\left(\frac{nRT}{P}\right)^\gamma = P^{1-\gamma} n^\gamma R^\gamma T^\gamma = const \longrightarrow P^{1-\gamma} T^\gamma = const$$

$$\Rightarrow \boxed{P T^{\frac{\gamma}{1-\gamma}} = const}$$