Riepilogo trasformazioni termodinamiche

Trasformazione	ΔU	Q	W
Isoterma $\Delta T=0$	$nc_v\Delta T=0$	$nRT\ln\left(\frac{V_f}{V_i}\right)=nRT\ln\left(\frac{P_i}{P_f}\right)$	$nRT\ln\left(\frac{V_f}{V_i}\right)=nRT\ln\left(\frac{P_i}{P_f}\right)$
Isobara $\Delta P=0$	$nc_v\Delta T=\frac{Pc_v}{R}\Delta V$	$nc_p\Delta T=\frac{c_p}{R}P\Delta V=\left(1+\frac{c_v}{R}\right)P\Delta V$	$P\Delta V=nR\Delta T$
Isocora $\Delta V=0$	$nc_v\Delta T=\frac{Vc_v}{R}\Delta P$	$nc_v\Delta T=\frac{Vc_v}{R}\Delta P$	0
Adiabatica $Q=0$	$nc_v\Delta T=\frac{P_fV_f-P_iV_i}{\gamma-1}=\frac{c_v}{R}(P_fV_f-P_iV_i)$	0	$-nc_v\Delta T=\frac{P_iV_i-P_fV_f}{\gamma-1}=\frac{c_v}{R}(P_iV_i-P_fV_f)$
Espansione libera adiabatica (non quasistatica)	0	0	0

$$\begin{matrix} T_i & & T_f \\ P_i & \longrightarrow & P_f \\ V_i & & V_f \end{matrix} \qquad \begin{matrix} \Delta T\equiv T_f-T_i \\ \Delta P\equiv P_f-P_i \\ \Delta V\equiv V_f-V_i \end{matrix}$$

Alcune relazioni utili per determinare le uguaglianze di sopra

- Isoterma $\quad Q=W \quad$ da $\Delta U=Q-W$
- Isobara $\quad c_p=c_v+R$
- Isocora $\quad W=P\Delta V=0$
- Adiabatica $\quad PV^\gamma=\text{const} \quad TV^{\gamma-1}=\text{const} \quad PT^{\frac{\gamma}{1-\gamma}}=\text{const} \quad \gamma=\frac{c_p}{c_v} \quad \gamma-1=\frac{R}{c_v}$
- Espansione libera adiabatica $\quad \Delta T=0$

$$dU=nc_vdT=\delta Q-PdV$$

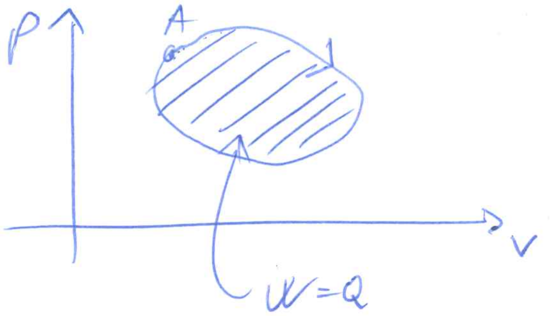
$$\delta Q=\begin{cases} nc_vdT+PdV \\ nc_pdT-VdP \end{cases}$$

Trasformazioni cicliche

"Banalmente" primo principio si applica anche se stato iniziale = stato finale

$\Rightarrow \boxed{\Delta U = 0}$

$\Rightarrow \boxed{Q = W}$



curva chiusa PV

Senso orario $\rightarrow$ W positivo (<P> più alta in espansione, più bassa compressione)

Senso antiorario $\rightarrow$ W negativo

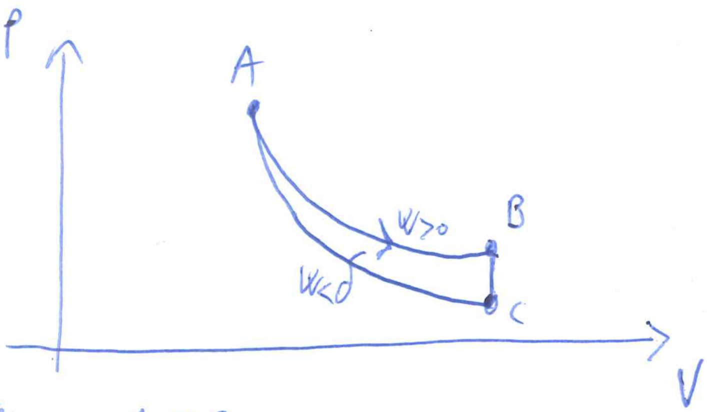
Esempio: IT-IC-AD

$P_A, V_A, T_A \xrightarrow{\text{isoterma quasistatica}} P_B, V_B, T_B = T_A$

isocora quasistatica

adiabatica

$P_C, V_C = V_B, T_C$



Isoterma $A \rightarrow B$

$\Delta U_{AB} = 0$ [dato che $T_A = T_B$ e $U = U(T)$]

$W_{AB} = nR \ln\left(\frac{P_A}{P_B}\right) T_A \qquad Q_{AB} = W_{AB}$ $\qquad Q_{AB}, W_{AB} > 0$

Isocora $B \rightarrow C$

$\Delta U_{BC} = n c_V (T_C - T_B) = n c_V (T_C - T_A)$ $\qquad T_C = \frac{P_C V_C}{nR} < T_A = \frac{P_A V_A}{nR}$

$W_{BC} = 0$ $\qquad T_B = \frac{P_B V_B}{nR}$

$Q_{BC} = \Delta U_{BC}$ $\qquad T_C < T_A, T_B$

Adiabatica C→A

$\Delta U_{CA} = nc_V(T_A - T_C)$

$W_{CA} = -\Delta U_{CA}$ $\quad Q_{CA} = 0$ $\qquad W<0\,,\ \Delta U>0$

→ nota: serve scelta particolare di R affinché $P_A V_A^{\gamma} = P_C V_C^{\gamma}$

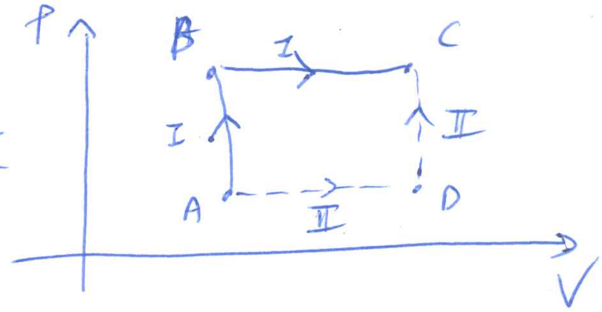
Lungo il ciclo

$\Delta U = 0 + nc_V(T_C - T_A) + nc_V(T_A - T_C) = 0$! Come dovrebbe essere

$W = nRT_A \ln\left(\frac{P_A}{P_B}\right) + \cancel{0}\, nc_V(T_A - T_C)$

$Q = nRT_A \ln\left(\frac{P_A}{P_B}\right) + nc_V(T_C - T_A) + 0 = W$! Come dovrebbe essere

Esempio 2: IB-IC / IC-IB



Considieriamo prima A→B→C

Isocora A→B

$\frac{P_B}{P_A} = \alpha > 1$ $\qquad T_A = \frac{P_A V_A}{nR} \rightarrow T_B = \frac{P_B V_B}{nR} = \alpha T_A$

$\Delta U_{AB} = nc_V \Delta T_{AB} = nc_V T_A(\alpha - 1)$

$W_{AB} = 0$ (in quanto $\Delta V = 0$) $\qquad Q = nc_V \Delta T_{AB} = nc_V T_A(\alpha - 1) = \Delta U_{AB} > 0$

~~Isocora~~ **Isobara B→C**

$\frac{V_C}{V_B} = \beta > 1$ $\qquad P_C = \frac{nRT_C}{V_C}$ $\qquad P_B = \frac{nRT_B}{V_B} \rightarrow T_C = \beta T_B$

$Q_{BC} = nc_P(T_C - T_B) = nc_P T_B(\beta - 1) = nc_P T_A \alpha(\beta - 1)$

$W_{BC} = P_B(V_C - V_B) = nR(T_C - T_B) = nRT_B(\beta - 1) = nRT_A\alpha(\beta - 1)$

$\Delta U_{BC} = nc_V(T_C - T_B) = nc_V T_B(\beta - 1) = nc_V T_A \alpha(\beta - 1) = Q_{BC} - W_{BC}$ in quanto $c_V = c_P - R$

Complessivamente

$$\Delta U_{AC}^{I} = \Delta U_{AB} + \Delta U_{BC} = n c_v T_A(\alpha - 1) + n c_v T_A \alpha(\beta - 1) = \cancel{n c_v T_A \alpha (\beta-1)}$$

$$= n c_v T_A(\alpha\beta - 1)$$

$$Q_{AC}^{I} = n c_v T_A(\alpha - 1) + n c_p T_A \alpha(\beta - 1) = n T_A \left[c_v(\alpha - 1) + c_p \alpha(\beta - 1)\right]$$

$$W_{AC}^{I} = 0 + nRT_A\alpha(\beta - 1) = nRT_A\alpha(\beta - 1)$$

Ora consideriamo $A \to D \to C$

~~Isobara $A \to D$~~

$$\cancel{\Delta U_{AD} = n c_v (T_A - T_A) = n c_v T_A(\alpha - 1)} \qquad \cancel{T_D = \frac{p_D V_D}{nR} = \frac{p_B V_D}{nR} = \frac{p_B}{p}} \qquad \cancel{T_D = \frac{p_D V_A}{nR}\,\frac{p_A}{p_B}}$$

Isobara $A \to D$

$$\Delta U_{AD} = n c_v (T_D - T_A) = \qquad T_D = \frac{p_D V_D}{nR} = \frac{p_A \beta V_A}{nR} = \beta T_A$$

$$= n c_v T_A(\beta - 1)$$

$$Q_{AD} = n c_p (T_D - T_A) = n c_p T_A(\beta - 1) \qquad c_p = c_v + R$$

$$W_{AD} = p_A(V_D - V_A) = nR(T_D - T_A) = nRT_A(\beta - 1) = Q_{AD} - \Delta U_{AD}$$

Isocora $D \to C$

$$T_C = \frac{p_C V_C}{nR} = \frac{\alpha p_D V_D}{nR} = \alpha T_D$$

$$\Delta U_{DC} = n c_v (T_C - T_D) = n c_v T_D(\alpha - 1) = n c_v T_A \beta(\alpha - 1)$$

$$W_{DC} = 0$$

$$Q_{DC} = \Delta U_{DC} = n c_v T_A \beta(\alpha - 1)$$

Complessivamente

$$\Delta U_{AC}^{II} = \Delta U_{AD} + \Delta U_{DC} = n c_v T_A(\beta - 1) + n c_v T_A \beta(\alpha - 1) = n c_v T_A(\alpha\beta - 1) = \Delta U_{AC}^{I}!$$

$$Q_{AC}^{II} = n c_p T_A(\beta - 1) + n c_v T_A \beta(\alpha - 1) = n T_A\left[c_p(\beta - 1) + c_v \beta(\alpha - 1)\right] \neq Q_{AC}^{I}!$$

$W_{AC}^{II} = W_{AD} + W_{DC} = nRT_A(\beta-1) + 0 = nRT_A(\beta-1) \neq W_{AC}^{I}$!

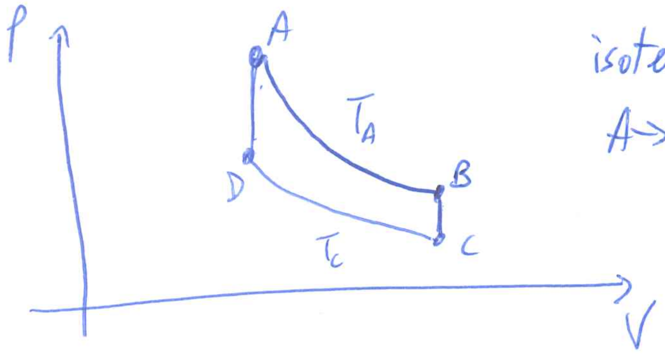
$\Delta U_{AC}^{I} = \Delta U_{AC}^{II}$ come dovrebbe essere in quanto U funzione di stato (dipende solo ~~dal percorso~~ dai punti iniziale e finale)

mentre $Q_{AC}^{I} \neq Q_{AC}^{II}$, $W_{AC}^{I} \neq W_{AC}^{II}$, confermando che Q e W non sono funzioni di stato, ma dipendono dal percorso seguito

Non si può parlare di Q, W posseduti, ma solo scambiati dal sistema

③ Esempio: Ciclo di Stirling

Brevettato nel 1816 dal reverendo Stirling



isoterma → isocora → isoterma → isocora

$A \to B$ | $B \to C$ raffreddamento | $C \to D$ | $D \to A$ riscaldamento

Isoterma $A \to B$

$\Delta U_{AB} = 0$ $\quad Q_{AB} = nRT_A \ln\left(\frac{V_B}{V_A}\right) = nRT_A \ln\left(\frac{P_A}{P_B}\right) = W_{AB}$

Isocora $B \to C$

$W_{BC} = 0$ $\quad \Delta U_{BC} = nc_v(T_C - T_A) = Q_{BC}$

Isoterma $C \to D$

$\Delta U_{CD} = 0$ $\quad Q_{CD} = nRT_C \ln\left(\frac{V_D}{V_C}\right) = nRT_C \ln\left(\frac{P_C}{P_D}\right) = W_{CD}$

Isocora $D \to A$

$W_{DA} = 0$ $\quad \Delta U_{DA} = nc_v(T_A - T_C) = Q_{DA}$

In totale: $\Delta U_{A\to A} = 0 + nc_v(T_C - T_A) + 0 + nc_v(T_A - T_C) = 0$ $\qquad W_{A\to A} = nRT_A \ln\left(\frac{P_A}{P_B}\right) + 0 + nRT_C \ln\left(\frac{P_C}{P_D}\right) + 0$

$Q_{A\to A} = nRT_A \ln\left(\frac{P_A}{P_B}\right) + nc_v(\cancel{T_C - T_A}) + nRT_C \ln\left(\frac{P_C}{P_D}\right) + nc_v(\cancel{T_A - T_C}) = nRT_A \ln\left(\frac{P_A}{P_B}\right) + nRT_A \ln\left(\frac{P_C}{P_D}\right) = W_{A\to A}$!