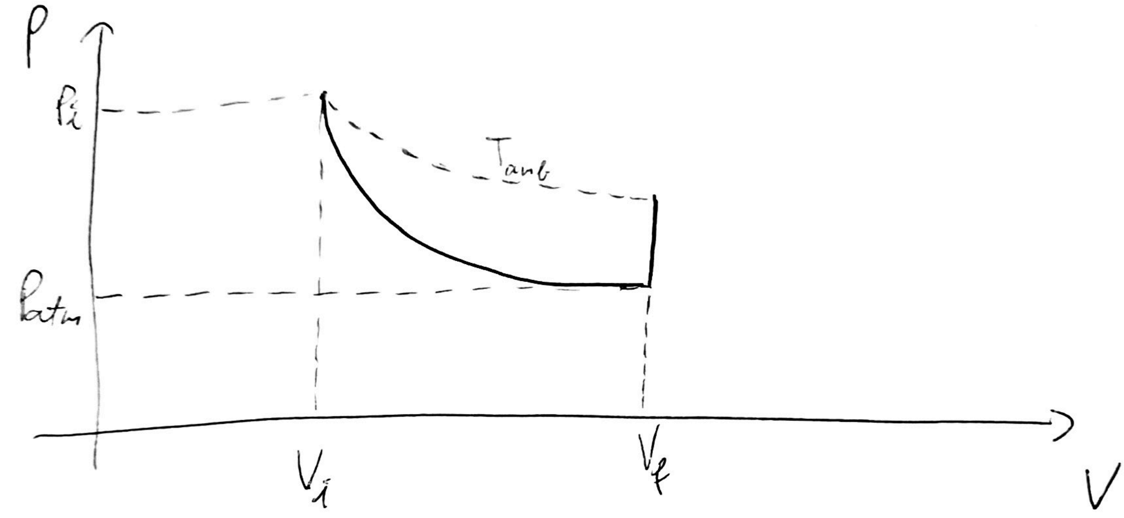
Esercizi

$\boxed{1}$

Metodo di Clément-Désormes per misurare γ

$$P_i, V_i, T_{amb} \xrightarrow[\text{adiabatica}]{\text{Espansione}} P_{atm}, V_f, T<T_{amb} \xrightarrow[\text{isocora}]{\text{Espansione}} P_f, V_f, T_{amb}$$

$P_i > P_{atm}$



$P_iV_i^{\gamma} = P_{atm}V_f^{\gamma}$ (1) adiabatica

$P_iV_i = P_fV_f$ (2) dato che $T_i = T_f = T_{amb}$

$$\Rightarrow \frac{P_i}{P_{atm}} \overset{(1)}{=} \cancel{\left(\frac{V_f}{V_i}\right)} \left(\frac{V_f}{V_i}\right)^{\gamma} \overset{(2)}{=} \left(\frac{P_i}{P_f}\right)^{\gamma}$$

$$\Rightarrow \ln\left(\frac{P_i}{P_{atm}}\right) = \ln\left[\left(\frac{P_i}{P_f}\right)^{\gamma}\right] = \gamma \ln\left(\frac{P_i}{P_f}\right) \rightarrow \gamma = \frac{\ln(P_i/P_{atm})}{\ln(P_i/P_f)}$$

In pratica P si misura con manometri/barometri $\rightarrow \begin{cases} P_i = \rho g h_i \\ P_{atm} = \rho g h_{atm} \\ P_f = \rho g h_f \end{cases}$

$\cancel{\gamma = \ln}$

$P_i = \rho g h_i = \rho g h_{atm}\left(1 + \frac{h_i}{h_{atm}}\right)$

$P_f = \rho g h_f = \cancel{\rho g h_f (1+} \rho g h_{atm}\left(1 + \frac{h_f}{h_{atm}}\right)$

$$\gamma = \frac{\ln(P_i/P_{atm})}{\ln(P_i/P_f)} = \frac{\ln\left(1+\frac{h_i}{h_{atm}}\right)}{\ln\left(1+\frac{h_i}{h_{atm}}\right) - \ln\left(1+\frac{h_f}{h_{atm}}\right)} \simeq \frac{h_i}{h_i - h_f}$$

$\ln(1+x) \simeq x$ per $x \to 0$

$$\gamma \simeq \frac{h_i}{h_i - h_f}$$

$\to$ facilmente misurabili!

$\boxed{2}$ Gas ideale, n moli. Trasformazione

$P(V) = \alpha V^2$ $\quad \alpha > 0$ da V_i a $V_f = \beta V_i$ $\quad \beta > 1$

Calcolare:

- ΔU
- W — dal primo principio
- Q — da $\delta Q = n c_p \delta T - V dP$

$$P_i = \alpha V_i^2 \longrightarrow P_f = \alpha V_f^2 = \alpha(\beta V_i)^2 = \beta^2(\alpha V_i^2) = \beta^2 P_i$$

- ΔU

$\Delta U = n c_v \Delta T$

$$PV = nRT \to T_i = \frac{P_i V_i}{nR} = \frac{\alpha V_i^3}{nR}$$

$P_f = \beta^2 P_i$
$V_f = \beta V_i$

$$T_f = \frac{P_f V_f}{nR} = \frac{\beta^3 P_i V_i}{nR} = \beta^3 T_i$$

$T_i = \frac{P_i V_i}{nR}$

$$\Delta U = n c_v \Delta T = n c_v (T_f - T_i) = n c_v (\beta^3 T_i - T_i) = n c_v (\beta^3 - 1) T_i$$

- W

$$W = \int_{V_i}^{V_f} dV\, P(V) = \int_{V_i}^{V_f} dV\, \alpha V^2 = \frac{\alpha}{3} V^3 \Big|_{V_i}^{V_f} = \frac{\alpha}{3}(V_f^3 - V_i^3) = \frac{\alpha}{3}(\beta^3 V_i^3 - V_i^3) = \frac{\alpha}{3}(\beta^3 - 1) V_i^3$$

• Q

Dal primo principio

$$Q = \Delta U + W = nc_v(\beta^3-1)T_i + \frac{\alpha}{3}(\beta^3-1)V_i^3 = \cancel{n}c_v(\beta^3-1)\frac{\alpha V_i^3}{\cancel{n}R} + \frac{\alpha}{3}(\beta^3-1)V_i^3 =$$

$$= \alpha c_v(\beta^3-1)\frac{V_i^3}{R} + \frac{\alpha(\beta^3-1)}{3}V_i^3 = \boxed{\alpha(\beta^3-1)V_i^3\left[\frac{c_v}{R}+\frac{1}{3}\right]}$$

Da $\delta Q = nc_p dT - VdP$

$$\cancel{\delta Q} \quad Q = \int_i^f \delta Q = \int_{T_i}^{T_f} dT\, nc_p - \int_{P_i}^{P_f} dP\, V(P)$$

Come scrivere dT e dP?

$$T = \frac{PV}{nR} = \frac{\alpha V^2 V}{nR} = \frac{\alpha V^3}{nR} \rightarrow dT = d\left(\frac{\alpha V^3}{nR}\right) = \frac{\alpha}{nR}d(V^3) = \frac{3\alpha V^2 dV}{nR}$$

$$P = \alpha V^2 \rightarrow dP = d(\alpha V^2) = \alpha d(V^2) = 2\alpha V dV$$

$$Q = \int \delta Q = \int dT\, nc_p - \int dP\, V = \int dV \frac{3\alpha V^2}{\cancel{n}R}\cancel{n}c_p - \int dV\, 2\alpha V V =$$

$$= \int dV\left[\frac{3\alpha c_p}{R}V^2 - 2\alpha V^2\right] = \int dV\left(\frac{3\alpha c_p}{R} - 2\alpha\right)V^2 = \left(\frac{3\alpha c_p}{R} - 2\alpha\right)\int_{V_i}^{V_f} dV\, V^2 =$$

$$= \left(\frac{3\alpha c_p}{R} - 2\alpha\right)\frac{V^3}{3}\Bigg|_{V_i}^{V_f} = \alpha\left(\frac{c_p}{R} - \frac{2}{3}\right)\left[V_f^3 - V_i^3\right] = \alpha\left(\frac{c_p}{R} - \frac{2}{3}\right)\left[\beta^3 V_i^3 - V_i^3\right] =$$

$$= \alpha(\beta^3-1)V_i^3\left[\frac{c_p}{R} - \frac{2}{3}\right]$$

$$\overset{?}{=} \alpha(\beta^3-1)V_i^3\left[\frac{c_v}{R} + \frac{1}{3}\right] \quad \text{se} \quad \frac{c_p}{R} - \frac{2}{3} = \frac{c_v}{R} + \frac{1}{3}$$

Verifichiamo

$$\frac{c_p}{R} - \frac{2}{3} = \frac{c_v + R}{R} - \frac{2}{3} = \frac{c_v}{R} + 1 - \frac{2}{3} = \frac{c_v}{R} + \frac{1}{3} \quad ✓$$

QED

$\boxed{3}$ ~~DF8,1~~ Controllare $\begin{cases} \text{numeri alla fine} \\ \text{unità SI} \\ \text{unità tornano} \end{cases}$

- Rame $V_i = 1 dm^3$ $T_i = 20°C \longrightarrow T_f = 30°C$

$P = P_{atm} = 1 atm$

$\rho = 8.94 \frac{g}{cm^3}$

$c_p^{(m)} = 380 \frac{J}{kg \cdot K}$

$\beta = 51 \times 10^{-6} K^{-1}$

Calcolare:

a) Q fornito, $\Delta U, \Delta H$

b) Tempo necessario assumendo Q fornito tramite resistore elettrico $P = 500W$

c) Altezza da cui bisogna buttare il blocco assumendo calore sviluppato nell'urto tutto assorbito dal bloccketto

a) $H = U + PV$

$dH = dU + d(PV) = dU + P\delta V + VdP = \delta Q - \cancel{P dV} + \cancel{P dV} + VdP = \delta Q + VdP$

Quindi a pressione costante, $dP = 0$, $dH = \delta Q$

$\Delta H = Q = m c_p^{(m)} \Delta T = \rho V c_p^{(m)} \Delta T$

- $V = 1 dm^3 = 10^{-3} m^3$

$\Delta T = 10°C = 10K$ (buona norma prima trasformare tutte le T in K

$\rho = 8.94 \frac{g}{cm^3} = 8.94 \frac{10^{-3} kg}{10^{-6} m^3} = 8.94 \times 10^3 \frac{kg}{m^3}$

$Q = \rho V c_p^{(m)} \Delta T = 8.94 \times 10^3 \times 10^{-3} \times 3.8 \times 10^2 \times 10 \; \frac{\cancel{kg}}{\cancel{m^3}} \cancel{m^3} \frac{J}{\cancel{kg}\, \cancel{K}} \cancel{K} \simeq 3.4 \times$

$= \Delta H$ (dato che $P = const$)

$\Delta U = Q - W$ $W = P\Delta V = P\beta V \Delta T = 10^5 \times 5.1 \times 10^{-5} \times 10^{-3} \times 10 \; Pa \frac{1}{K} m^3 K =$

$\simeq 0.05 \; Pa\, m^3 = 0.05 \frac{N}{m^2} m^3 = 0.05 N \cdot m = 0.05 J \ll Q$

$\Rightarrow \Delta U = Q - W \simeq Q \simeq 3.4 \times 10^4 J$

b) $P = \dfrac{Q}{\Delta t} \rightarrow \Delta t = \dfrac{Q}{P} = \dfrac{3.4\times10^4\,J}{500\,W} \simeq 68\,s$ (~ 1 min)

e se invece volessi riscaldare H_2O con lo stesso resistore?

$c_p = 4186 \dfrac{J}{kg \cdot k}$

$\rho = 1 \dfrac{g}{cm^3}$

$$\Delta t = \frac{Q}{P} = \frac{\rho V c_p^{(c.p)} \Delta T}{P}$$

$$\Delta t_{H_2O} = \frac{Q_{H_2O}}{P} = \frac{\rho_{H_2O} V c_{p,H_2O} \Delta T}{P} = \left(\frac{\rho_{H_2O}}{\rho_{Cu}}\right)\left(\frac{c_{p,H_2O}}{c_{p,Cu}}\right) \Delta t_{Cu} =$$

$$\Delta t_{Cu} = \frac{Q_{Cu}}{P} = \frac{\rho_{Cu} V c_{p,Cu} \Delta T}{P}$$

$$= \frac{1}{8.94} \frac{4186}{380} t_{Cu} \simeq 1.23\, t_{Cu} \simeq 84\,s \;(\sim 1.5\,min)$$

c) Conservazione dell'energia (assumendo tutta energia cinetica assorbita come calore)

Potenziale		Cinetica		Calore
mgh	$\longrightarrow$	$\frac{1}{2}mv^2$	$\longrightarrow$	Q

$$Q = mgh \longrightarrow h = \frac{Q}{mg} = \frac{3.4\times10^4\,J}{8.94\,kg \cdot 9.81\,\frac{m}{s^2}} \simeq 387 \frac{J}{N} = 387\,m$$

DP8.1.14

$\boxed{4}$ $n = 0.4$ mol gas monoatomico ideale

$V_i = 1\,l$ $\quad P_i = 10$ bar $\quad$ Calcolare $\begin{cases} a)\ V_f, T_f \\ b)\ W \end{cases}$

$\downarrow$ Espansione adiabatica

$P_f = 1$ bar

a) $C_v = \frac{3}{2}R \quad C_p = \frac{5}{2}R \rightarrow \gamma = \frac{C_p}{C_v} = \frac{5}{3}$

$PV^\gamma = \text{const} \qquad P_iV_i^\gamma = P_fV_f^\gamma \Rightarrow V_f = \left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}} V_i = 10^{\frac{3}{5}} \ell =$

$\simeq 3.98 \ell = 3.98 \times 10^{-3} m^3$

$T_f = \frac{P_fV_f}{nR}$ (da $PV = nRT$) $\qquad P_f = 1 \text{ bar} = 10^5 Pa$

$T_f = \frac{10^5 \times 3.98 \times 10^{-3}}{0.4 \times 8.31} \frac{Pa \cdot m^3}{mol \cdot \frac{J}{K \cdot mol}} \simeq 119.7 \frac{Pa \cdot m^3}{\frac{J}{K}} = 119.7 \frac{J}{\frac{J}{K}} = 119.7 K$

b) $\Delta U = Q - W \Rightarrow W = Q - \Delta U = -\Delta U = -nC_v\Delta T =$

$Q = 0$ / adiabatica

$= nC_v(T_i - T_f) = \frac{3}{2}Rn(T_i - T_f)$

$T_f \simeq 119.7 K$

$T_i = \frac{P_iV_i}{nR} = \frac{10^6 Pa \times 10^{-3} m^3}{0.4 mol \cdot 8.31 \frac{J}{K \cdot mol}} \simeq 300.8 K$

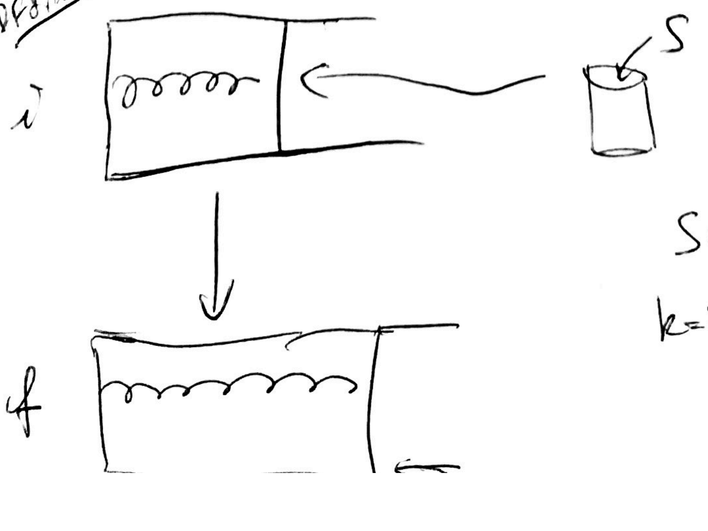
$W = -\Delta U = \frac{3}{2}nR(T_i - T_f) = \frac{3}{2} \times 0.4 \times 8.31 \times (300.8 - 119.7) \; mol \frac{J}{K \cdot mol} K \simeq$

$\simeq 903 J$

$\Delta U \simeq -903 J$ (gas si raffredda)

$\boxed{5}$ DEFORMA

Gas ideale monoatomico



molla a riposo

$S = 1 dm^2 \qquad P_i = P_{atm} = 10^5 Pa$

$k = 2 \times 10^4 \frac{N}{m} \qquad V_i = 1 dm^3 \qquad T_i = 300 K$

$V_f = 1.2 dm^3$

Calcolare $\begin{cases} a) & n \\ b) & dP/dV \quad e \quad P(V) \\ c) & P_f, T_f \\ d) & Q, W, \Delta U \end{cases}$

a) $PV = nRT$

$$\rightarrow \quad n = \frac{PV}{RT} = \frac{10^5 \times 10^{-3}}{8.31 \times 300} \simeq 0.04 \text{ mol}$$

b) $P = P_{atm} + \frac{F}{S} = P_{atm} + \frac{kx}{S} \Longrightarrow dP = d\left(P_{atm} + \frac{kx}{S}\right) = d\left(\frac{kx}{S}\right) = \frac{k}{S} dx$

legge di Hooke

$V = S \cdot x \quad \rightarrow dV = d(Sx) = S dx$

$$\rightarrow \frac{dP}{dV} = \frac{k/S \, dx}{S dx} = \frac{k}{S^2} \qquad dP = \frac{k}{S^2} dV \quad \text{relazione lineare}$$

c) lunghezza iniziale $1 \text{ dm} = 10^{-1} \text{ m}$] dato che $V_i = 1 \text{ dm}^3$

lunghezza finale $1.2 \text{ dm} = 1.2 \times 10^{-1} \text{ m}$] $V_f = 1.2 \text{ dm}^3$

$V = S \cdot \ell \qquad S = 1 \text{ dm}^2$

$x = 0.2 \text{ dm} = 2 \times 10^{-2} \text{ m}$

$\Delta V = 0.2 \text{ dm}^3 = 0.2 \times 10^{-3} \text{ m}^3 = 2 \times 10^{-4} \text{ m}^3$

$S = 1 \text{ dm}^2 = 10^{-2} \text{ m}^2$

$$P_f = P_i + \int dV \frac{dP}{dV} = P_{atm} + \int dV \frac{k}{S^2} = P_{atm} + \frac{k}{S^2} \Delta V = 10^5 \text{ Pa} + \frac{2 \times 10^4 \times 2 \times 10^{-4}}{(10^{-2})^2} \text{ Pa} = 1.4 \times 10^5 \text{ Pa}$$

$PV = nRT$

$$\Rightarrow T_f = \frac{P_f V_f}{nR} = \frac{1.4 \times 10^5 \times 1.2 \times 10^{-3}}{0.04 \times 8.31} \text{ K} \simeq 505 \text{ K}$$

d) $\Delta U = n c_v \Delta T = \frac{3}{2} nR \Delta T = \frac{3}{2} nR (T_f - T_i) = \frac{3}{2} \times 0.06 \times 8.31 \times (505-300) J \simeq 102 J$

$c_v = \frac{3}{2} R$

$dW = P dV$

$\Rightarrow W = \int dV P(V) = \int dV [P_{atm} + P_{molla}] = P_{atm} \int dV + \int dV \frac{K}{S} \cancel{x} =$

$= P_{atm} \int dV + \int dx\, \cancel{S} \frac{K}{\cancel{S}} x = P_{atm} \underbrace{\Delta V}_{2 dm^3} + \int_0^{x_f} dx\, Kx =$

$= P_{atm} \Delta V + K \frac{x^2}{2} \Big|_0^{x_f} = P_{atm} \Delta V + K \frac{x_f^2}{2}$

$x_f = 0.2 dm = 2 \times 10^{-2} m$

$W = 10^5 Pa \times 2 \times 10^{-4} m^3 + 2 \times 10^4 \frac{N}{m} \times \frac{(2 \times 10^{-2} m)^2}{2} = 24 J$

$\Delta U = Q - W$

$\Rightarrow Q = \Delta U + W \simeq 102 J + 24 J = 126 J$

$\boxed{6}$ PF8.2

Cilindro con pareti diatermiche verticale

^4He $\quad m = 8g \quad T = 27°C$

a) Espansione isobara fino a $V \to 2V$. Calcolare $W, Q, \Delta U, \Delta H$

Assumiamo $P = P_{atm} = 10^5 Pa$ (se non è esplicitamente detto)

$T_i = 27°C \simeq 300K$

$P_i = P_{atm} = 10^5 Pa$

$n = \frac{\text{massa totale}}{\text{massa molare}} = \frac{8 g}{4 g/mol} = 2 mol$

$V_i = \frac{nRT_i}{P_i} = \frac{2 \times 8.31 \times 300}{10^5} m^3 \simeq 4.986 \times 10^{-2} m^3$

Stato iniziale

$P_f = P_i = 10^5\,Pa$

$V_f = 2V_i = 9.972 \times 10^{-2}\,m^3$

$T_2 = \dfrac{P_f V_f}{nR} = \dfrac{2P_iV_i}{nR} = 2T_i = 600\,K$

$n = 2\,mol$

Stato finale

~~[illegible]~~

$W = P\Delta V = P(V_2 - V_1) = 10^5 \times (9.972 - 4.986) \times 10^{-2} \simeq 4986\,J > 0$

gas fa lavoro

$Q = nc_p\Delta T = \dfrac{5}{2} nR\Delta T = \dfrac{5}{2} \times 2 \times 8.31 \times (600-300) \simeq 12465\,J$

^4He monoatomico

$$\Delta U = \begin{cases} Q - W = 12465\,J - 4986\,J = 7479\,J \\ nc_v\Delta T = \dfrac{3}{2} nR\Delta T = \dfrac{3}{2} \times 2 \times 8.31 \times (600-300) = 7479\,J \end{cases}$$

$H = U + PV$

Sappiamo già che dovremmo trovare $\Delta H = Q_p = Q = 12465\,J$

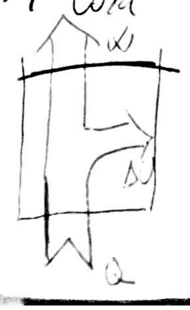
$P_iV_i = 10^5\,Pa \times 4.986 \times 10^{-2}\,m^3 = 4986\,J$

$P_fV_f = 10^5\,Pa \times \cancel{4.986}\ 9.972 \times 10^{-2}\,m^3 = 9972\,J$

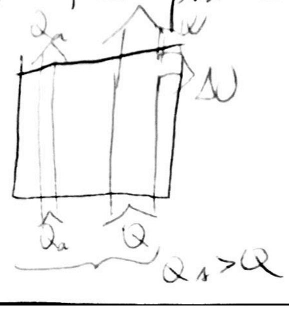
$\Delta(PV) = P_fV_f - P_iV_i = 9972\,J - 4986\,J = 4986\,J$

$\Delta H = \Delta U + \Delta(PV) = 7479\,J + 4986\,J = 12465\,J = Q\,!$

b) Cosa succede se c'è attrito fra pistone e cilindro?



Senza attrito



Con attrito

Con attrito ~~ha~~ a parità di lavoro utile $P\Delta V$

- il gas deve fare più lavoro
- la trasformazione non è reversibile
- il gas deve assorbire più calore

F: risultante delle forze d'attrito
S: sezione cilindro

$$W_{tot} = W + W_a = \left(P + \frac{F}{S}\right)\Delta V$$ di cui
- lavoro utile $P\Delta V$
- dissipato in calore $\frac{F}{S}\Delta V$

$$Q_{tot} = \Delta U + W_{tot} = \underbrace{\Delta U + W}_{Q} + W_a = Q + W_a$$

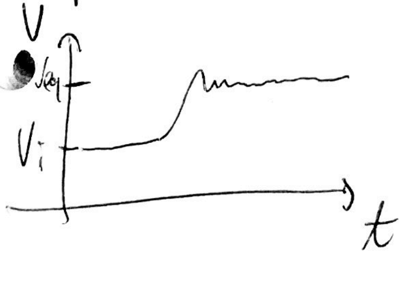
Quindi il sistema deve assorbire Q_a in più, che poi però cederà tutto all'ambiente!

Quantità netta di calore ceduta dall'ambiente: $Q + Q_a - Q_a = Q$ come prima!

c) Cilindro spostato rapidamente da $T_i = 300K$ a $T_f = 600K$
$P_i = 1 atm \rightarrow P_f = 1 atm$

Come cambiano i risultati?

Moto del pistone: accelerazione esterna, poi oscillazioni intorno alla posizione di equilibrio



stesso stato finale del caso reversibile

$W = P\Delta V = 4986 J$

$\Delta U = 7479 J$

$Q = 12465 J$