Mathematical Methods for Physicists
Hand-in problems I

Due September 6, 2017 before 15:00. This is a hard deadline since we want to have the possibility to discuss hand-in problems at the lecture. We will not accept hand-ins after this deadline. Email your nice LaTeX solutions to Sunny (*sunny.vagnozzi@fysik.su.se*). **Remember to write your email address on the top of the first page of your solutions**! Scanned solutions will be accepted if, and only if, the quality is good enough to be read and corrected. Please submit solutions in electronic form in PDF format only.
[**Total: 12 points** [+2 **bonus points**]]

1. Consider a boat travelling in water with an instantaneous velocity v. The boat is subject to a resistance proportional to v^n, where $n > 2$. That is, the boat's equation of motion reads:

$$m\frac{dv}{dt} = -kv^n \,, \tag{1}$$

where m is the mass of the boat, t denotes time, and k is a constant.

- Solve for the velocity $v(t)$ subject to the initial condition $v(t = 0) = v_0$, with $v_0 > 0$.
- Show that the solution you determined for v satisfies the differential equation [Eq. (1)]
- Hence by integrating $v(t)$, determine the position of the boat $x(t)$, subject to the initial condition $x(t = 0) = x_0$, with $x_0 > 0$
- Plot $v(t)$ and $x(t)$.

[**3 points**]

2. Consider a chain of two decays of radioactive nuclei which we will call A and B respectively. Specifically, A decays to B which then decays to the stable nucleus C (the latter is unimportant for the purpose of the exercise). The two have concentrations given by N_A and N_B respectively. The evolution of N_A is determined by the usual universal law of radioactive decay, i.e. by:

$$\frac{dN_A}{dt} = -\lambda_A N_A \,, \tag{2}$$

where λ_A is its decay constant. The evolution of N_B is instead determined by the following equation:

$$\frac{dN_B}{dt} = -\lambda_B N_B + \lambda_A N_A \,, \tag{3}$$

where λ_B is its decay constant.

- Solve for $N_B(t)$ subject to the initial conditions $N_A(t = 0) = N_0$, with $N_0 > 0$, and $N_B(t = 0) = 0$
- Show that the solution you determined for N_B satisfies the differential equation [Eq. (3)]
- What happens if B is a stable nucleus, i.e. if $\lambda_B = 0$? Show what happens to your solution in this case and comment on why you should have expected this
- *For bonus points*: Now consider C, the decay product of B, decaying as well, to a stable nucleus D. Determine the evolution equation of $N_C(t)$ and solve it subject to the same initial conditions as previously, as well as $N_C(t = 0) = 0$

[**3+2 points**]

3. • Solve the following differential equation:

$$\frac{d^3x}{dt^3} - 2\frac{d^2x}{dt^2} - \frac{dx}{dt} + 2x = 0\,, \tag{4}$$

subject to the initial conditions $x''(0) = 6$, $x'(0) = 2$, $x(0) = 3$.

• Provide a general **real** solution for the following differential equation (that is, no complex quantities should appear in your solution):

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0\,. \tag{5}$$

[**3 points**]

4. Using the Frobenius approach, obtain the general solution of the following equation:

$$3x\frac{d^2y}{dx^2} + (3x+1)\frac{dy}{dx} + y = 0\,. \tag{6}$$

If the solution cannot be represented analytically, give the four largest terms in the Frobenius expansion in the limit $x \to 0$
[**3 points**]