Mathematical Methods for Physicists
Hand-in problems II

Due September 13, 2017 before 15:00. This is a hard deadline since we want to have the possibility to discuss hand-in problems at the lecture. We will not accept hand-ins after this deadline. Email your nice LaTeX solutions to Sunny (*sunny.vagnozzi@fysik.su.se*). **Remember to write your email address on the top of the first page of your solutions**! Scanned solutions will be accepted if, and only if, the quality is good enough to be read and corrected. Please submit solutions in electronic form in PDF format only.

[**Total: 14 points**]

1. Write the stationary Schrödinger equation:

$$-\frac{\hbar^2}{2\,m}\nabla^2\,\Psi + V\,\Psi = E\,\Psi\,, \tag{1}$$

in spherical coordinates, explicitly ($\hbar$, m, and E are constants, $V = -\frac{1}{r}$ is a function of the distance to the origin only). The solution can be written as $\Psi(r,\theta,\phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$. Find the solution for $\Phi(\phi)$ using separation of variables and set up the separated equations for the $R(r)$ and $\Theta(\theta)$ dependence (no need to solve the latter two).
[**3 points**]

2. The heat equation is a differential equation describing the distribution of a temperature field $\psi(x,t)$ over time, and reads:

$$\frac{\partial\psi}{\partial t} = a^2\frac{\partial^2\psi}{\partial x^2}\,. \tag{2}$$

Now consider the temperature field $\psi(x,t)$ of a rod of length L, with position on the rod given by the variable x. The ends of the rod at $x=0$ and $x=L$ are kept at the respective temperatures $\psi(x=0,t)=\pi$ and $\psi(x=L,t)=0$ for all times t. Initially, $\psi(x,t=0)=0$ for $0\le x\le L$. Solve for $\psi(x,t)$.

Hints: Try to find a general solution on the form $\psi(x,t)=u(x)+v(x,t)$, where the function $u(x)$ satisfies the stationary one-dimensional heat equation and initial conditions $u(0)=\pi$ and $u(L)=0$. The function $v(x,t)$ instead satisfies the time-dependent one-dimensional heat equation with initial conditions $v(0,t)=0$, $v(L,t)=0$ and $v(x,0)=-u(x)$.
[**4 points**]

3.
   - Using the Green's function method, solve the differential equation:

$$\frac{d^2y}{dx^2} = e^x\,, \tag{3}$$

   subject to the boundary conditions $y(0)=0$, $y(1)=0$
   [**2 points**]

   - Now do the same for the differential equation:

$$\frac{d^2y}{dx^2} = \cosh(\pi x)\,, \tag{4}$$

   with the same boundary conditions $y(0)=0$, $y(1)=0$
   [**2 points**]

   - Now do it for the simpler differential equation:

$$\frac{d^2y}{dx^2} = x^5\,, \tag{5}$$

with boundary conditions $y(0) = 0$, $y(5) = 0$. However, this time, together with solving it using the Green's function method, solve the differential equation explicitly by integrating twice and imposing the appropriate boundary conditions. Show that the solution you recover matches the one obtained using the Green's function method.
[**3 points**]