Mathematical Methods for Physicists
Hand-in problems IV

Due September 28, 2016 before 15:00. Because Sunny (*sunny.vagnozzi@fysik.su.se*) will be away 19-28 September, physical solutions will **not** be accepted. Please make sure you email your solutions in PDF format, either typed or scanned. Remember to put your email on top.
[**Total: 12 points**]

1. Determine the nature of the singularities of each of the following complex functions f and g and evaluate the residues ($z \in \mathbb{C}, b > 0$)

   (a)
   $$f : \mathbb{C} \longrightarrow \mathbb{C};\ f : z \mapsto \frac{1}{(z^2 + b^2)^2}$$
   [**2 points**]

   (b)
   $$g : \mathbb{C} \longrightarrow \mathbb{C};\ g : z \mapsto \frac{1}{\sinh(\pi z)}$$
   (you might want to use the de l'Hopital rule)
   [**2 points**]

2. (a) Explain how the residue is related to the Laurent series.
   [**1 point**]

   (b) Expand the complex function k
   $$k : \mathbb{C} \longrightarrow \mathbb{C},\ k : z \mapsto \frac{e^z}{z^3}$$
   in its Laurent series around $z = 0$ and find the residue.
   [**2 points**]

   (c) Find the residue of the complex function f
   $$f : \mathbb{C} \longrightarrow \mathbb{C},\ f : z \mapsto \frac{1}{z(1 + e^z)}$$
   in $z = 0$.
   [**1 point**]

3. Recall Cauchy's residue theorem. For $x \in \mathbb{R}$, calculate the following integrals using extension to the complex plane:

   (a)
   $$I_1 := \int_0^\infty dx\ \frac{x \sin(x)}{1 + x^2}\ ;$$
   [**2 points**]

   (b)
   $$I_2 := \int_0^\infty dx\ \frac{x \sin(x)}{x^2 - a^2}\ ,$$
   where $a > 0$.
   [**2 points**]

   [*Hint*: think of $x \sin(x)$ as the imaginary part of ze^{iz}, for appropriate values of z.]