Mathematical Methods for Physicists
Hand-in problems IV

Due September 27, 2017 before 15:00. This is a hard deadline since we want to have the possibility to discuss hand-in problems at the lecture. We will not accept hand-ins after this deadline. Email your nice LaTeX solutions to Sunny (*sunny.vagnozzi@fysik.su.se*). **Remember to write your email address on the top of the first page of your solutions**! Scanned solutions will be accepted if, and only if, the quality is good enough to be read and corrected. Please submit solutions in electronic form in PDF format only.

[**Total: 12 points**]

1. (a) Consider the following complex function:

$$f : \mathbb{C} \longrightarrow \mathbb{C};\ f : z \mapsto \frac{\pi \cot(\pi z)}{z(z+1)}\,. \tag{1}$$

Evaluate its residues in $z = 0$ and $z = -1$.
[**2 points**]

(b) Now consider the following complex function:

$$g : \mathbb{C} \longrightarrow \mathbb{C};\ g : z \mapsto z^{11} e^{\frac{1}{z^3}}\,. \tag{2}$$

What type of singularity is $z = 0$? Evaluate the residue in $z = 0$.
[**2 points**]

2. (a) Explain how the residue is related to the Laurent series. Then expand the complex function k:

$$k : \mathbb{C} \longrightarrow \mathbb{C},\ k : z \mapsto \frac{e^z}{z^3}$$

in its Laurent series around $z = 0$ and find the residue.
[**2 points**]

(b) Evaluate the 4 residues of the complex function g:

$$g : \mathbb{C} \longrightarrow \mathbb{C},\ g : z \mapsto \frac{1}{z^4 + 5z^2 + 6}$$

[**2 points**]

3. Recall Cauchy's residue theorem. For $x \in \mathbb{R}$, calculate the following integrals using extension to the complex plane:

(a)

$$I_1 := \int_{-\infty}^{\infty} dx\ \frac{1}{x^2 + x + 1}\,;$$

[**2 points**]

(b)

$$I_2 := \int_{0}^{\infty} dx\ \frac{x \sin(x)}{x^2 + 121}\,,$$

[**2 points**]