Mathematical Methods for Physicists
Hand-in problems V

Due October 5, 2016 before 15:00. It is preferable if you email your solutions to Sunny (*sunny.vagnozzi@fysik.su.se*), either LaTeX'd or scanned (PDF format), or alternatively bring them in person to A5:1078. Submission through third parties, especially in the case of physical copies, is highly discouraged as there is the risk of them not reaching the intended receiver. Please remember to put your email on top.

[**Total: 12 points**]

1. Evaluate the following series:

$$S = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^4}$$

[**3 points**]
[*Hint*: write the series as $S = \sum_{n=1}^{\infty} \frac{1}{n^4} - \sum_{n=1}^{\infty} \frac{1}{(2n)^4} = \frac{15}{16} \sum_{n=1}^{\infty} \frac{1}{n^4}$]

2. Evaluate the following series:

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n}{1-4n^2}$$

[**3 points**]

3. The Legendre polynomials can be represented by the so called *Rodrigues'* formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n \,.$$

Using this relation, show that the Legendre polynomials take the value 1 at $x = 1$, i.e. $P_n(1) = 1$.
Hint: Write (x^2-1) as $(x+1)(x-1)$, then use Leibnitz's rule to differentiate n times:

$$\frac{d^n}{dx^n}\left[A(x)B(x)\right] = \sum_{s=0}^{n} \binom{n}{s} \frac{d^{n-s}}{dx^{n-s}} A(x) \frac{d^s}{dx^s} B(x) \,.$$

[**3 points**]

4. Show that, for integer $n \geq 0$

(a)

$$\int_0^{\infty} x^{2n+1}\, e^{-ax^2}\, dx = \frac{n!}{2a^{n+1}} \,.$$

(b)

$$\int_0^{\infty} x^{2n}\, e^{-ax^2}\, dx = \frac{\Gamma(n+\frac{1}{2})}{2a^{n+1/2}} = \frac{(2n-1)!!}{2^{n+1}a^n} \sqrt{\frac{\pi}{a}} \,.$$

These Gaussian integrals are of major importance in statistical mechanics.
[**3 points**]