Mathematical Methods for Physicists
Hand-in problems VI

Due October 11, 2017 before 15:00. This is a hard deadline since we want to have the possibility to discuss hand-in problems at the lecture. We will not accept hand-ins after this deadline. Email your nice LaTeX solutions to Sunny (*sunny.vagnozzi@fysik.su.se*). **Remember to write your email address on the top of the first page of your solutions**! Scanned solutions will be accepted if, and only if, the quality is good enough to be read and corrected. Please submit solutions in electronic form in PDF format only.

[**Total: 12 points**]

1. Evaluate the following series:

   (a)

$$\sum_{n=1}^{\infty} \frac{1}{(4n+2)^2}$$

   (b)

$$\sum_{n=2}^{\infty} \frac{1}{(2n+1)^4}$$

   [**6 points**]

2. Although you might not necessarily have seen the Chebyshev polynomials of the second kind in class, these are a set of polynomials usually denoted by $U_n(x)$, which solve the Chebyshev differential equation:

$$(1-x^2)\frac{d^2U_n(X)}{dx^2} - 3x\frac{dU_n(x)}{dx} + n(n+2)U_n(x) = 0\,.$$

   However, you know what a generating function is, so it suffices for you to know that the generating function for the Chebyshev polynomials of the second kind is:

$$g(x,t) = \frac{1}{1-2xt+t^2}\,.$$

   By differentiating the generating function and manipulating the result, show that the Chebyshev polynomials of the second kind satisfy the following recursion relation:

$$U_{n-1}(x) - 2xU_n(x) + U_{n+1}(x) = 0\,.$$

   [**3 points**]

3. For $z \in \mathbb{R}$, $m \in \mathbb{N}$, and $J_m(z)$ Bessel functions of the first kind, derive the Jacobi-Anger expansion of a plane wave in a series of cylindrical waves:

$$e^{iz\cos\theta} = \sum_{m=-\infty}^{\infty} i^m J_m(z) e^{im\theta}\,.$$

   [**3 points**]