Mathematical Methods for Physicists
Hand-in problems VIII

Due November 4, 2016 before 15:00. Email your solutions to Sunny (*sunny.vagnozzi@fysik.su.se*), either LaTeX'd or scanned (PDF format), or if necessary photographed. Because of research travel there might be some delay in marking the handin.

[**Total: 14 points**]

1. Let $\tilde{f}(s) \equiv \mathcal{L}\left[f(t)\right]$ denote the Laplace transform, $k \neq 0$. Find the inverse Laplace transform satisfying: $\mathcal{L}^{-1}\left[\mathcal{L}\left[f(t)\right]\right] = f(t)$ for

$$\tilde{f}(s) = \frac{s}{s^2 - k^2}$$

   (a) using a partial fraction expansion and tabulated backtransforms.
   [**2 points**]

   (b) using the Bromwich integral.
   [**2 points**]

2. (a) Find the Fourier transform of $f(x)$ defined as $f(x) = 1 - |x/2|$ for $-2 \leq x \leq 2$, $f(x) = 0$ elsewhere. [**2 points**]

   (b) Evaluate the following integral using the Parseval relation:

$$\int_{-\infty}^{\infty} dk \, \left(\frac{\sin k}{k}\right)^4$$

   [**1 point**]

3. Find the Laplace transform $\tilde{f}(s) \equiv \mathcal{L}\left[f(t)\right]$ of the following functions, with $a \geq 0$:

   (a)

$$f(t) \equiv \frac{\sin(at)}{at}$$

   [**2 points**]

   (b)

$$f(t) \equiv \frac{\sinh(at)}{at}$$

   [**2 points**]

4. The one-dimensional neutron diffusion equation with a plane source is:

$$-D\frac{d^2\Phi(x)}{dx^2} + K^2 D\Phi(x) = Q\delta(x)\,,$$

   where $\Phi(x)$ is the neutron flux, $Q\delta(x)$ is the plane source at $x = 0$, D and K^2 are real constants. Solve the above equation for $\Phi(x)$ by Fourier transforming.
   [**3 points**]