Mathematical Methods for Physicists
Hand-in problems VIII

Due November 2, 2017 before 15:00. This is a hard deadline since we want to have the possibility to discuss hand-in problems at the lecture. We will not accept hand-ins after this deadline. Email your nice LaTeX solutions to Sunny (*sunny.vagnozzi@fysik.su.se*). **Remember to write your email address on the top of the first page of your solutions**! Scanned solutions will be accepted if, and only if, the quality is good enough to be read and corrected.
Please submit solutions in electronic form in PDF format only.

[**Total: 14 points**]

1. Consider the following differential equation:

$$f(t) - \frac{d^2 f}{dt^2} = 2(1+2t)e^t\,, \quad t \geq 0\,, \quad f(0) = \frac{df}{dt}(0) = 1\,. \tag{1}$$

   - Find $\tilde{g}(s) \equiv \mathcal{L}\left[g(t)\right]$, where $g(t) = 2(1+2t)e^t$ is the right-hand side of the above equation, and $\mathcal{L}$ denotes the Laplace transform.
   - Using the result above, find the Laplace transform of $f(t)$, $\tilde{f}(s) \equiv \mathcal{L}\left[f(t)\right]$, by Laplace-transforming the left-hand side of the above equation.
   - If you have done things correctly, it's likely you have ended up with an expression for $\tilde{f}(s)$ which is either already decomposed in partial fractions, or is decomposable. Use this decomposition together with tabulated backtransforms to obtain $f(t)$, and verify that it satisfies both the differential equations and the initial conditions.
   - Now obtain $f(t)$ from $\tilde{f}(s)$ using the Bromwich integral, and verify that your result matches the one you obtained using the partial fraction decomposition and the tabulated backtransforms.

   [**4 points**]

2. Verify that:

$$\mathcal{L}\left[f(t) = \frac{\sin(at)}{at}\right](s) = \frac{1}{a}\cot^{-1}\left(\frac{s}{a}\right)\,, \tag{2}$$

$$\mathcal{L}\left[f(t) = \frac{\sinh(at)}{at}\right](s) = \frac{1}{a}\coth^{-1}\left(\frac{s}{a}\right)\,, \tag{3}$$

   where $a \neq 0$. You might find the relation $\coth^{-1}(x) = 1/2 \times \ln((x+1)/(x-1))$ useful.
   [**4 points**]

3. The one-dimensional neutron diffusion equation with a plane source is:

$$-D\frac{d^2\Phi(x)}{dx^2} + K^2 D\Phi(x) = Q\delta(x)\,,$$

   where $\Phi(x)$ is the neutron flux, $Q\delta(x)$ is the plane source at $x = 0$ (with $\delta(x)$ the Dirac delta), and D and K^2 are real constants. Solve the above equation for $\Phi(x)$ by Fourier transforming.
   [**3 points**]

4. Consider the following differential equation:

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + y(x) = 4e^{-x}\,, \quad y(0) = 2\,, \quad \frac{dy}{dx}(0) = -1\,. \tag{4}$$

   Solve the above equation by the method of Laplace transforms, and verify that your solution satisfies the original equation and the the boundary conditions.
   [**3 points**]