Fysikum, Stockholm University
Cosmology & Astroparticle physics course, 2017

Assignment: **set 1**. **Deadline: April 18th**

P 1.1 The solution of Einstein's equations in an empty Universe with a cosmological constant Λ was first found by de Sitter solving:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda g_{\mu\nu}.$$

The $\mu\nu = 00$ and $\mu\nu = 11$ solutions give the Friedmann and acceleration (Eqns (4.7) and (4.8) in the book).

Find the $\mu\nu = 22$ and $\mu\nu = 33$ solutions and show that they do *not* provide any independent information.

P 1.2 Consider an *empty* ($\rho_M = \rho_\gamma = \rho_\Lambda = 0$) *expanding* universe.

a) What is the spatial curvature (k) of such a universe?

b) Show that all components of $R_{\mu\nu}$ vanish.

c) Show that the luminosity distance in such a universe relates to *any* redshift as:

$$d_L = \frac{cz}{H_0}\left(1 + \frac{z}{2}\right)$$

P 1.3 A particle with momentum p has a de Broglie wavelength $\lambda = h/p$ that increases with the expansion of the universe as $\lambda \propto a$. The total energy density, ρ, of a gas of particles can be written as $\rho = nE$ where n is the number density of particles and

$$E = \sqrt{p^2c^2 + m^2c^4}\,,$$

is the energy per particle at any given time where we assume that all particles have the same mass, m, and momentum p.

a) Derive an expression of the equation-of-state parameter, w, for the gas as a function of the scale factor, a.

b) Calculate w in the relativistic, $p \to \infty$, and non-relativistic $p \to 0$, limits, respectively.