Fysikum, Stockholm University
Cosmology & Astroparticle physics course, 2017

## Assignment: **set 2**. **Deadline: May 16th**

P 2.1 In the present day universe the cosmic microwave background radiation is severely redshifted; its photons currently have energies of around $E_{\text{cmb}} \simeq 2 \times 10^{-4}\,\text{eV}$. Despite the low energies, the CMB is still an important source of opaqueness for high energy astronomy. The quoted energies are relative a 'rest frame', essentially the frame of the Earth; give your results in this frame.

(a) A high energy photon $\gamma$ is emitted at a cosmological distance. In its path towards a telescope at Earth, it encounters a CMB photon $\gamma_{\text{cmb}}$ and produces an electron-positron pair through the process: $\gamma + \gamma_{\text{cmb}} \to e^+ + e^-$. Calculate the threshold energy for this process. Also calculate the threshold energy if the encountered photon instead comes from starlight, with photon energy $E_{\text{sl}} \simeq 0.3\,\text{eV}$.

(b) A high energy proton $p$ is emitted at a cosmological distance. In its path towards a telescope at Earth the proton interacs with a CMB photon $\gamma_{\text{cmb}}$. The scatter excites the proton into a higher energy spin configuration with spin $\frac{3}{2}$, i.e. the proton is turned into a delta baryon: $\Delta^+$. The process is hence: $p + \gamma_{\text{cmb}} \to \Delta^+$. Calculate the threshold energy for this process. The mass of the delta baryon is $m_{\Delta^+} \simeq 1232\,\text{MeV}$.

P 2.2 The number of dark matter particles in the early Universe is governed by their rate of creation and annihilation through:

$$\frac{\mathrm{d}(a^3 n_\chi)}{\mathrm{d}t} = -\langle \sigma_A v \rangle \left[ n_\chi^2 - (n_\chi^{eq})^2 \right] a^3 .$$

Use this and

$$Y_\chi \equiv \frac{n_\chi}{s}; \quad x = \frac{m_\chi}{T}; \quad \Gamma_A = n_\chi^{eq} \langle \sigma_A v \rangle ,$$

to derive the main formula for the freeze-out of dark matter:

$$\frac{x}{Y_\chi^{eq}} \frac{dY_\chi}{dx} = -\frac{\Gamma_A}{H(x)} \left[ \left( \frac{Y_\chi}{Y_\chi^{eq}} \right)^2 - 1 \right] .$$

It might be useful to note that the freeze-out of dark matter is expected to happen during radiation domination.
(Hint: The conservation of entropy, $sa^3 = const$, implies that $s\dot{Y} = \dot{n}_\chi + 3Hn_\chi$. If this relation is used, show that it is indeed a consequence of entropy conservation.)

P 2.3 In FLRW cosmologies, we find that the observable universe must have been composed of multiple causally disconnected regions at the time of recombination. This is known as the Horizon Problem.

For this problem, we can assume that recombination happened instantaneously at redshift $z = 1090$ and that prior to recombination, matter and radiation behaved as single fluid with sound speed $v_s = c/\sqrt{3}$. After recombination, you can assume that there was no coupling between radiation and matter.

(a) We define conformal time, $\tau(t)$, according to

$$\tau(t) = \int_0^t \frac{dt'}{a(t')}.$$

Derive an analytical expression for the conformal time at which the CMB radiation was emitted. Why are you allowed to neglect curvature and cosmological constant terms in this derivation?

(b) The sound horizon represents the maximum distance that sound waves in the photon-baryon fluid could have propagated before the photons making up the cosmic microwave background were released. Use the results from part a) to estimate the size of the comoving sound horizon in units of $h^{-1}$ Mpc.

(c) Find an analytic expression for the angular size (in degrees) corresponding to this acoustic peak as seen at the present epoch ($t_0$), assuming a flat universe with $\{\Omega_\mathrm{m}, \Omega_\Lambda\} = \{1.0, 0.0\}$ and an open universe with $\{\Omega_\mathrm{m}, \Omega_\Lambda\} = \{0.3, 0.0\}$. Finally, numerically estimate this angle for our universe, $\{\Omega_\mathrm{m}, \Omega_\Lambda\} = \{0.3, 0.7\}$.

(d) Now, consider our observable universe (all of the volume within our horizon). Calculate the diameter of this spherical region at the time just after inflation ended, $t_\mathrm{end} \approx 10^{-34}\,\mathrm{s}$.